JEE-MAIN – JUNE, 2022

(Held On Tuesday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

Mathematics

Test Pattern : JEE-MAIN Maximum Marks : 120

Topic Covered: FULL SYLLABUS

Important instruction:

1. Use Blue / Black Ball point pen only.
2. There are three sections of equal weightage in the question paper **Physics, Chemistry** and **Mathematics** having 30 questions in each subject. Each paper have 2 sections A and B.
3. You are awarded +4 marks for each correct answer and –1 marks for each incorrect answer.
4. Use of calculator and other electronic devices is not allowed during the exam.
5. No extra sheets will be provided for any kind of work.

Name of the Candidate (in Capitals) _____

Father's Name (in Capitals) _____

Form Number : in figures _____

: in words _____

Centre of Examination (in Capitals): _____

Candidate's Signature: _____ Invigilator's Signature : _____

Rough Space

YOUR TARGET IS TO SECURE GOOD RANK IN JEE-MAIN

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $A=\{x\in R:|x+1|<2\}$ and $B=\{x\in R:|x-1|\geq 2\}$. Then which one of the following statements is **NOT** true ?

(A) $A-B=(-1,1)$ (B) $B-A=R-(-3,1)$

(C) $A\cap B=(-3,-1]$ (D) $A\cup B=R-[1,3)$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $A: x\in(-3, 1)$ $B: x\in(-\infty, -1]\cup[3,\infty)$

$B-A=(-\infty, -3]\cup[3,\infty)=R-(-3, 3)$

2. Let $a,b\in R$ be such that the equation $ax^2-2bx+15=0$ has a repeated root α. If α and β are the roots of the equation $x^2-2bx+21=0$, then $\alpha^2+\beta^2$ is equal to:

(A) 37 (B) 58

(C) 68 (D) 92

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $ax^2-2bx+15=0$

$2\alpha=\frac{2b}{a}, \alpha^2=\frac{15}{a}$

$\frac{\alpha}{2}=\frac{15}{2b}$

$\alpha=\frac{15}{b}$

$x^2-2bx+21=0$

$\left(\frac{15}{b}\right)^2-2b\left(\frac{15}{b}\right)+21=0$

$b^2=25$

$\alpha+\beta=2b,\ \alpha\beta=21$

$\alpha^2+\beta^2=4b^2-42$

$=58$

3. Let z_1 and z_2 be two complex numbers such that $\bar{z}_1=i\bar{z}_2$ and $\arg\left(\frac{z_1}{\bar{z}_2}\right)=\pi$. Then

(A) $\arg z_2=\frac{\pi}{4}$ (B) $\arg z_2=-\frac{3\pi}{4}$

(C) $\arg z_1=\frac{\pi}{4}$ (D) $\arg z_1=-\frac{3\pi}{4}$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $\bar{z}_1=i\bar{z}_2$

$z_1=-iz_2$

$\arg\left(\frac{z_1}{\bar{z}_2}\right)=\pi$

$\arg\left(-i\frac{z_2}{\bar{z}_2}\right)=\pi$ $\arg(z_2)=\theta$

$-\frac{\pi}{2}+\theta+\theta=\pi$

$2\theta=\frac{3\pi}{2}$

$\arg(z_2)=\theta=\frac{3\pi}{4}$, $\arg z_1=\frac{\pi}{4}$

4. The system of equations

$-kx+3y-14z=25$

$-15x+4y-kz=3$

$-4x+y+3z=4$

is consistent for all k in the set

(A) R (B) $R-\{-11,13\}$

(C) $R-\{13\}$ (D) $R-\{-11,11\}$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $\Delta = \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} = 121 - k^2$

$\Delta \neq 0 \qquad k \in R - \{11, -11\}$ (Unique sol.)

If k = 11

$$\Delta_z = \begin{vmatrix} -11 & 3 & 25 \\ -15 & 4 & 3 \\ -4 & 1 & 4 \end{vmatrix} \neq 0$$

No solution

If k = –11

$$\Delta_z = \begin{vmatrix} 11 & 3 & 25 \\ -15 & 4 & 3 \\ -4 & 1 & 4 \end{vmatrix} \neq 0$$

No solution

5. $\lim\limits_{x \to \frac{\pi}{2}} \left(\tan^2 x \left(\left(2\sin^2 x + 3\sin x + 4\right)^{\frac{1}{2}} - \left(\sin^2 x + 6\sin x + 2\right)^{\frac{1}{2}} \right) \right)$

is equal to

(A) $\frac{1}{12}$ (B) $-\frac{1}{18}$

(C) $-\frac{1}{12}$ (D) $-\frac{1}{6}$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol.

$$\lim_{x \to \frac{\pi}{2}} \tan^2 x \left[\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2} \right] =$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\tan^2 x\,[\sin^2 x - 3\sin x + 2]}{\sqrt{9} + \sqrt{9}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\tan^2 x(\sin x - 1)(\sin x - 2)}{6}$$

$$= \frac{1}{6} \lim_{x \to \frac{\pi}{2}} \tan^2 x(1 - \sin x)$$

$$= \frac{1}{6} \lim_{x \to \frac{\pi}{2}} \frac{\sin^2 x\,(1 - \sin x)}{(1 - \sin x)(1 + \sin x)} = \frac{1}{12}$$

6. The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

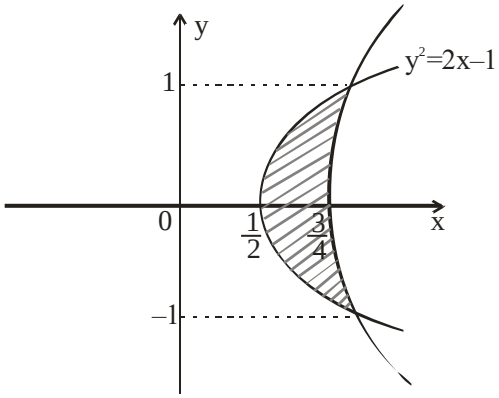
(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. Required area = $2\int_0^1 \left(\frac{y^2 + 3}{4} - \frac{y^2 + 1}{2} \right) dy$

$$= 2\int_0^1 \frac{1 - y^2}{4} dy = \frac{1}{2} \left| y - \frac{y^3}{3} \right|_0^1 = \frac{1}{3}$$



7. The coefficient of x^{101} in the expression

$(5 + x)^{500} + x(5 + x)^{499} + x^2(5 + x)^{498} + ...x^{500}$,

x > 0, is

(A) ${}^{501}C_{101}(5)^{399}$ (B) ${}^{501}C_{101}(5)^{400}$

(C) ${}^{501}C_{100}(5)^{400}$ (D) ${}^{500}C_{101}(5)^{399}$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $(5 + x)^{500} + x(5 + x)^{499} + x^2(5 + x)^{498} + ... + x^{500}$

$$= \frac{(5 + x)^{501} - x^{501}}{(5 + x) - x} = \frac{(5 + x)^{501} - x^{501}}{5}$$

$\Rightarrow$ coefficient x^{101} in given expression

$$= \frac{{}^{501}C_{101} 5^{400}}{5} = {}^{501}C_{101} 5^{399}$$

8. The sum $1+2\cdot3+3\cdot3^2+.....+10\cdot3^9$ is equal to

(A) $\frac{2\cdot3^{12}+10}{4}$ (B) $\frac{19\cdot3^{10}+1}{4}$

(C) $5\cdot3^{10}-2$ (D) $\frac{9\cdot3^{10}+1}{2}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $S=1\cdot3^0+2\cdot3^1+3\cdot3^2+.....+10.3^9$

$3S=1\cdot3^1+2.3^2.....+9\times3^9+10\times3^{10}$

$-2S=(1\cdot3^0+3^1+3^2.....3^9)-10.3^{10}$

$S=5\times3^{10}-\left(\frac{3^{10}-1}{4}\right)$

$S=\frac{20.3^{10}-3^{10}+1}{4}=\frac{19.3^{10}+1}{4}$

9. Let P be the plane passing through the intersection of the planes $\vec{r}\cdot\left(\hat{i}+3\hat{j}-\hat{k}\right)=5$ and $\vec{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)=3$, and the point $(2,1,-2)$. Let the position vectors of the points X and Y be $\hat{i}-2\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+2\hat{k}$ respectively. Then the points

(A) X and X + Y are on the same side of P

(B) Y and Y – X are on the opposite sides of P

(C) X and Y are on the opposite sides of P

(D) X + Y and X – Y are on the same side of P

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $P_1+\lambda P_2=0$

$\Rightarrow(x+3y-z-5)+\lambda(2x-y+z-3)=0$

$(2,1,-2)$ lies on this plane

$\therefore\lambda=1\Rightarrow$ plane is $3x+2y-8=0$

10. A circle touches both the y-axis and the line x + y = 0. Then the locus of its center is

(A) $y=\sqrt{2}x$ (B) $x=\sqrt{2}y$

(C) $y^2-x^2=2xy$ (D) $x^2-y^2=2xy$

Official Ans. by NTA (D)

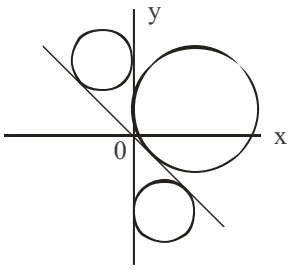
Allen Ans. (D)

Sol. Let (h, k) is centre of circle

$\left|\frac{h-k}{\sqrt{2}}\right|=|h|$

$k^2-h^2+2hk=0$

$\therefore$ Equation of locus is $y^2-x^2+2xy=0$



11. Water is being filled at the rate of $1\,cm^3/sec$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm^2/sec) at which the wet conical surface area of the vessel increases is

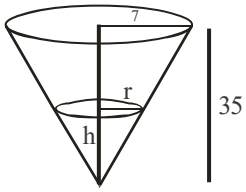
(A) 5 (B) $\frac{\sqrt{21}}{5}$

(C) $\frac{\sqrt{26}}{5}$ (D) $\frac{\sqrt{26}}{10}$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. From figure $\frac{r}{h}=\frac{7}{35}\Rightarrow h=5r$



Given $\frac{dV}{dt}=1\Rightarrow\frac{d}{dt}\left(\frac{\pi r^2h}{3}\right)=1$

$\Rightarrow\frac{d}{dt}\left(\frac{5\pi}{3}r^3\right)=1\Rightarrow r^2\frac{dr}{dt}=\frac{1}{5\pi}$

Let wet conical surface area = S

$=\pi r\ell=\pi r\sqrt{h^2+r^2}$

$=\sqrt{26}\pi r^2\Rightarrow\frac{dS}{dt}=2\sqrt{26}\pi r\frac{dr}{dt}$

When h = 10 then r = 2 $\Rightarrow\frac{dS}{dt}=\frac{2\sqrt{26}}{10}$

12. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x} dx$, $n \in N$, then

(A) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference -2

(B) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference 2

(C) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

(D) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference -2

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $b_n = \int_0^{\pi/2} \frac{1 + \cos 2nx}{\sin x} dx$

$b_{n+1} - b_n = \int_0^{\pi/2} \frac{\cos^2 (n+1)x - \cos^2 nx}{\sin x} dx$

$= \int_0^{\pi/2} \frac{-\sin(2n+1)x \sin x}{\sin x} dx$

$= \left(\frac{\cos(2n+1)x}{2n+1} \right)_0^{\pi/2} = \frac{-1}{2n+1}$

$\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with c.d.$= -2$

13. If $y = y(x)$ is the solution of the differential equation $2x^2 \frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(e) = \frac{e}{3}$, then $y(1)$ is equal to

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) 3

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $\frac{dy}{dx} - \frac{y}{x} = -\frac{3}{2}\left(\frac{y}{x}\right)^2$ $y = vx$

$\frac{dv}{v^2} = -\frac{3dx}{2x}$

$-\frac{1}{v} = -\frac{3}{2} \ell n|x| + C$

$-\frac{x}{y} = \frac{-3}{2} \ell n|x| + C$

$x = e,\ y = \frac{e}{3}$

$C = -\frac{3}{2}$

When $x = 1$, $y = \frac{2}{3}$

14. If the angle made by the tangent at the point (x_0, y_0) on the curve $x = 12(t + \sin t \cos t)$, $y = 12(1 + \sin t)^2, 0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to

(A) $6\left(3 + 2\sqrt{2}\right)$ (B) $3\left(7 + 4\sqrt{3}\right)$

(C) 27 (D) 48

Official Ans. by NTA (C)

Allen Ans. (3)

Sol. $\frac{dy}{dx} = \frac{2(1 + \sin t) \times \cos t}{1 + \cos 2t}$

$\Rightarrow \frac{2(1 + \sin t)\cos t}{2\cos^2 t} = \sqrt{3}$

$\Rightarrow t = \frac{\pi}{6}, y_0 = 27$

15. The value of $2\sin(12°) - \sin(72°)$ is :

(A) $\frac{\sqrt{5}\left(1 - \sqrt{3}\right)}{4}$ (B) $\frac{1 - \sqrt{5}}{8}$

(C) $\frac{\sqrt{3}\left(1 - \sqrt{5}\right)}{2}$ (D) $\frac{\sqrt{3}\left(1 - \sqrt{5}\right)}{4}$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $\sin 12° + \sin 12° - \sin 72°$

$= \sin 12° - 2\cos 42° \sin 30°$

$= \sin 12° - \sin 48°$

$= -2\cos 30° \sin 18°$

$= -2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{5}-1}{4}$

$= \frac{\sqrt{3}}{4}(1-\sqrt{5})$

16. A biased die is marked with numbers 2,4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is

(A) $\frac{7}{2^{11}}$ (B) $\frac{7}{2^{12}}$

(C) $\frac{3}{2^{10}}$ (D) $\frac{13}{2^{12}}$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $P(n) = \frac{1}{n}$

$P(2) = \frac{1}{2}$ $P(8) = \frac{1}{8}$

$P(4) = \frac{1}{4}$ $P(16) = \frac{1}{16}$

$P(32) = \frac{2}{32}$

Possible cases

16, 16, 16 and 32, 8, 8

Probability $= \frac{1}{16^3} + \frac{2}{32} \times \frac{1}{8} \times \frac{1}{8} \times 3 = \frac{13}{16^3}$

17. The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$ is logically equivalent to

(A) $p \Rightarrow q$ (B) $q \Rightarrow p$

(C) $\sim(p \Rightarrow q)$ (D) $\sim(q \Rightarrow p)$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $\sim p \vee q \equiv p \rightarrow q$

$\sim q \wedge p \equiv \sim (p \rightarrow q)$

Negation of $\sim (p \rightarrow q) \rightarrow (p \rightarrow q)$

is $\sim (p \rightarrow q) \wedge (\sim (p \rightarrow q))$ i.e. $\sim (p \rightarrow q)$

18. If the line $y = 4 + kx$, $k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is :

(A) $\frac{3}{2}$ (B) $\frac{26}{9}$

(C) $\frac{5}{2}$ (D) $\frac{23}{6}$

Official Ans. by NTA (C)

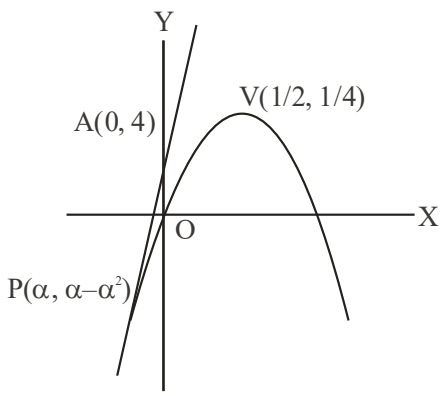
Allen Ans. (C)

Sol. Slope of tangent at P = Slope of line AP

$y'|_P = 1 - 2\alpha = \frac{\alpha - \alpha^2 - 4}{\alpha}$

Solving $\alpha = -2 \Rightarrow P(-2, -6)$

Slope of PV = $\frac{5}{2}$



19. The value of $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ is equal to

(A) $-\frac{\pi}{4}$ (B) $-\frac{\pi}{8}$

(C) $-\frac{5\pi}{12}$ (D) $-\frac{4\pi}{9}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $\tan^{-1}\left[\dfrac{\cos\left(4\pi-\frac{\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right] \Rightarrow \tan^{-1}\left(\dfrac{\cos\frac{\pi}{4}-1}{\sin\frac{\pi}{4}}\right)$

$\tan^{-1}\left(\dfrac{1-\sqrt{2}}{1}\right)=-\dfrac{\pi}{8}$

20. The line $y=x+1$ meets the ellipse $\dfrac{x^2}{4}+\dfrac{y^2}{2}=1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to

(A) 20 (B) 12

(C) 11 (D) 8

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. Ellipse $x^2+2y^2=4$

Line y = x + 1

Point of intersection

$x^2+2(x+1)^2=4$

$3x^2+4x-2=0$

$|x_1-x_2|=\dfrac{\sqrt{40}}{3}$

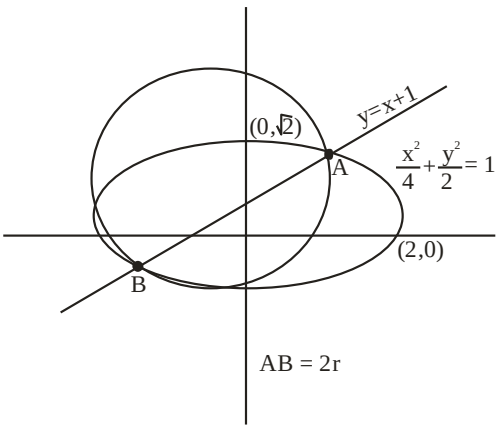
$AB=2r=|x_1-x_2|\sqrt{1+m^2}$,

m is slope of given line

$AB=\dfrac{\sqrt{40}}{3}\sqrt{1+1}$

$2r=\dfrac{\sqrt{80}}{3}\Rightarrow r=\dfrac{\sqrt{80}}{6}$

$(3r)^2=\left(3\times\dfrac{\sqrt{80}}{6}\right)^2=\dfrac{80}{4}=20$



SECTION-B

1. Let $A=\begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}$. Then the number of elements in the set

$\{(n,m): n,m\in\{1,2,.....,10\}$ and $nA^n+mB^m=I\}$

is ____

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $A^2=A$ and $B^2=B$

Therefore equation $nA^n+mB^m=I$ becomes

nA + mB = I, which gives m = n = 1

Only one set possible

2. Let $f(x)=[2x^2+1]$ and $g(x)=\begin{cases}2x-3, & x<0\\ 2x+3, & x\ge 0\end{cases}$,

where [t] is the greatest integer $\le t$. Then, in the open interval (–1, 1), the number of points where fog is discontinuous is equal to _____

Official Ans. by NTA (62)

Allen Ans. (62)

Sol. $f(g(x))=\left[2g^2(x)\right]+1$

$=\begin{cases}\left[2(2x-3)^2\right]+1;\ x<0\\ \left[2(2x+3)^2\right]+1; x\ge 0\end{cases}$

$\therefore$ fog is discontinuous whenever $2(2x-3)^2$ or $2(2x+3)^2$ belongs to integer except x = 0.

$\therefore$ 62 points of discontinuity.

3. The value of b > 3 for which

$12\int_3^b \dfrac{1}{(x^2-1)(x^2-4)}dx=\log_e\left(\dfrac{49}{40}\right)$, is equal to

Official Ans. by NTA (6)

Allen Ans. (6)

Sol. $\frac{12}{3}\left[\int_3^b\left(\frac{1}{x^2-4}-\frac{1}{x^2-1}\right)dx\right]=\log\frac{49}{40}$

$\frac{12}{3}\left[\frac{1}{4}\ell n\left|\frac{x-2}{x+2}\right|-\frac{1}{2}\ell n\left|\frac{x-1}{x+1}\right|\right]_3^b=\log\frac{49}{40}$

$\ell n\frac{(b-2)(b+1)^2}{(b+2)(b-1)^2}=\ell n\frac{49}{50}$

$b = 6$

4. If the sum of the coefficients of all the positive even powers of x in the binomial expansion of $\left(2x^3+\frac{3}{x}\right)^{10}$ is $5^{10}-\beta\cdot 3^9$, then β is equal to _____

Official Ans. by NTA (83)

Allen Ans. (83)

Sol. $T_{r+1}={}^{10}C_r(2x^3)^{10-r}\left(\frac{3}{x}\right)^r$

$={}^{10}C_r 2^{10-r}3^r x^{30-4r}$

Put r = 0, 1, 2, ... 7 and we get $\beta = 83$

5. If the mean deviation about the mean of the numbers 1, 2, 3, ..., n, where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to _____

Official Ans. by NTA (21)

Allen Ans. (21)

Sol. Mean deviation about mean of first n natural numbers is $\frac{n^2-1}{4n}$

$\therefore n = 21$

6. Let $\vec{b}=\hat{i}+\hat{j}+\lambda\hat{k},\lambda\in R$. If $\vec{a}$ is a vector such that $\vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$ and $\vec{a}\cdot\vec{b}+21=0$, then $(\vec{b}-\vec{a})\cdot(\hat{k}-\hat{j})+(\vec{b}+\vec{a})\cdot(\hat{i}-\hat{k})$ is equal to

Official Ans. by NTA (14)

Allen Ans. (14)

Sol. $(\vec{a}\times\vec{b}).\vec{b}=0$

$\Rightarrow 13-1-4\lambda=0\Rightarrow\lambda=3$

$\Rightarrow\vec{b}=\hat{i}+\hat{j}+3\hat{k}\ \Rightarrow\ \vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$

$\Rightarrow\ (\vec{a}\times\vec{b})\times\vec{b}=(13\hat{i}-\hat{j}-4\hat{k})\times(\hat{i}+\hat{j}+3\hat{k})$

$\Rightarrow -21\vec{b}-11\vec{a}=\hat{i}-43\hat{j}+14\hat{k}$

$\Rightarrow\ \vec{a}=-2\hat{i}+2\hat{j}-7\hat{k}$

Now $(\vec{b}-\vec{a}).(\hat{k}-\hat{j})+(\vec{b}+\vec{a}).(\hat{i}-\hat{k})=14$

7. The total number of three-digit numbers, with one digit repeated exactly two times, is

Official Ans. by NTA (243)

Allen Ans. (243)

Sol. If 0 taken twice then ways = 9

If 0 taken once then ${}^9C_1\times 2=18$

If 0 not taken then ${}^9C_1\,{}^8C_1.3=216$

Total = 243

8. Let $f(x)=|(x-1)(x^2-2x-3)|+x-3,\ x\in R$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then m + M is equal to ____

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $f(x)=\begin{cases}(x^2-1)(x-3)+(x-3), x\in(0,1]\cup[3,4)\\ -(x^2-1)(x-3)+(x-3), x\in[1,3]\end{cases}$

$\Rightarrow f'(x)=\begin{cases}3x^2-6x,\ x\in(0,1)\cup(3,4)\\ -3x^2+6x+2,\ x\in(1,3)\end{cases}$

f(x) is non-derivable at x = 1 and x = 3

also f'(x) = 0 at $x=1+\sqrt{\frac{5}{3}}\ \Rightarrow\ m+M=3$

9. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x+\beta y=\lambda$, then $\lambda-\beta$ is equal to

Official Ans. by NTA (85)

Allen Ans. (85)

Sol. $e^2=1+\frac{b^2}{a^2}=\frac{25}{16}\Rightarrow\frac{b^2}{a^2}=\frac{9}{16}$ **.....(1)**

$A\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ satisfies $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow\frac{64}{5a^2}-\frac{144}{25b^2}=1$ **.....(2)**

Solving (1) & (2) $b=\frac{6}{5}$ $a=\frac{8}{5}$

Normal at A is $\frac{\sqrt{5}a^2x}{8}+\frac{5b^2y}{12}=a^2+b^2$

Comparing it $8\sqrt{5}x+\beta y=\lambda$

Gives $\lambda=100,\ \beta=15$

$\lambda-\beta=85$

10. Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and l_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to

Official Ans. by NTA (51)

Allen Ans. (51)

Sol. $8x+4\sqrt{2}y=1,\ z=0$

$\Rightarrow\frac{x-\frac{1}{8}}{1}=\frac{y-0}{-\sqrt{2}}=\frac{z-0}{0}=\lambda$

$-8x-6\sqrt{3}z=1,\ y=0$

$\Rightarrow\frac{x+\frac{1}{8}}{3\sqrt{3}}=\frac{y-0}{0}=\frac{z-0}{-4}$

$$\begin{vmatrix}\frac{1}{4} & 0 & 0\\ 1 & -\sqrt{2} & 0\\ 3\sqrt{3} & 0 & -4\end{vmatrix}=\sqrt{2}$$

$d=\frac{1}{\sqrt{51}}$

$\frac{1}{d^2}=51$