



## JEE-MAIN – JUNE, 2022

(Held On Tuesday 26<sup>th</sup> June, 2022)

TIME : 9 : 00 AM to 12 : 00 PM

### Mathematics

Test Pattern : JEE-MAIN

Maximum Marks : 120

Topic Covered: FULL SYLLABUS

#### Important instruction:

1. Use Blue / Black Ball point pen only.
2. There are three sections of equal weightage in the question paper **Physics, Chemistry** and **Mathematics** having 30 questions in each subject. Each paper have 2 sections A and B.
3. You are awarded +4 marks for each correct answer and -1 marks for each incorrect answer.
4. Use of calculator and other electronic devices is not allowed during the exam.
5. No extra sheets will be provided for any kind of work.

Name of the Candidate (in Capitals) \_\_\_\_\_

Father's Name (in Capitals) \_\_\_\_\_

Form Number : in figures \_\_\_\_\_

: in words \_\_\_\_\_

Centre of Examination (in Capitals): \_\_\_\_\_

Candidate's Signature: \_\_\_\_\_ Invigilator's Signature : \_\_\_\_\_

Rough Space

**YOUR TARGET IS TO SECURE GOOD RANK IN JEE-MAIN**

Corporate Office : **ALLEN Digital Pvt. Ltd.**, "One Biz Square", A-12 (a), Road No. 1, Indraprastha Industrial Area, Kota (Rajasthan) INDIA-324005

📞 +91-9513736499 | 📞 +91-7849901001 | 📩 wecare@allendigital.in | 🌐 www.allendigital.in

## FINAL JEE-MAIN EXAMINATION – JUNE, 2022

**(Held On Sunday 26<sup>th</sup> June, 2022)**

**TIME : 9 : 00 AM to 12 : 00 PM**

### MATHEMATICS

### TEST PAPER WITH SOLUTION

#### SECTION-A

1. Let  $f(x) = \frac{x-1}{x+1}$ ,  $x \in \mathbb{R} - \{0, -1, 1\}$ . If  $f^{n+1}(x) = f(f^n(x))$

for all  $n \in \mathbb{N}$ , then  $f^6(6) + f^7(7)$  is equal to:

- (A)  $\frac{7}{6}$       (B)  $-\frac{3}{2}$       (C)  $\frac{7}{12}$       (D)  $-\frac{11}{12}$

**Official Ans. by NTA (B)**

**Allen Ans. (B)**

**Sol.**  $f(x) = \frac{x-1}{x+1}$

$$\Rightarrow f^2(x) = f(f(x)) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} = -\frac{1}{x}$$

$$f^3(x) = f(f^2(x)) = f\left(-\frac{1}{x}\right) = \frac{x+1}{1-x}$$

$$\Rightarrow f^4(x) = f\left(\frac{x+1}{1-x}\right) = -\frac{1}{x}$$

$$\Rightarrow f^6(x) = -\frac{1}{x} \Rightarrow f^6(6) = -\frac{1}{8}$$

$$f^7(x) = \left(-\frac{1}{x}\right) = \frac{x+1}{1-x}$$

$$\Rightarrow f^7(7) = \frac{8}{-6} = -\frac{4}{3}$$

$$\therefore -\frac{1}{6} + -\frac{4}{3} = -\frac{3}{2}$$

2. Let  $A = \left\{ z \in C : \left| \frac{z+1}{z-1} \right| < 1 \right\}$

and  $B = \left\{ z \in C : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3} \right\}$ .

Then  $A \cap B$  is :

- (A) a portion of a circle centred at  $\left(0, -\frac{1}{\sqrt{3}}\right)$  that

lies in the second and third quadrants only

- (B) a portion of a circle centred at  $\left(0, -\frac{1}{\sqrt{3}}\right)$  that

lies in the second quadrant only

- (C) an empty set

- (D) a portion of a circle of radius  $\frac{2}{\sqrt{3}}$  that lies in

the third quadrant only

**Official Ans. by NTA (B)**

**Allen Ans. (B)**

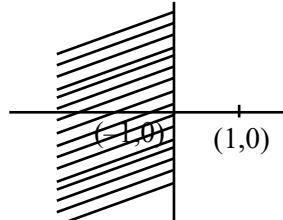
- Sol.** Set A

$$\Rightarrow \left| \frac{z+1}{z-1} \right| < 1$$

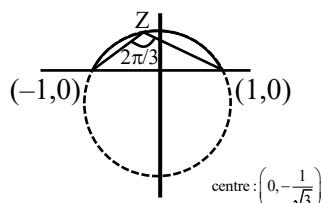
$$\Rightarrow |z+1| < |z-1|$$

$$\Rightarrow (x+1)^2 + y^2 < (x-1)^2 + y^2$$

$$\Rightarrow x < 0$$



Set B



$$\Rightarrow \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right) = \frac{2\pi}{3}$$

$$\Rightarrow x^2 + y^2 + \frac{2y}{\sqrt{3}} - 1 = 0$$

$A \cap B$

$$\Rightarrow \text{Centre } \left(0, -\frac{1}{\sqrt{3}}\right)$$

3. Let A be a  $3 \times 3$  invertible matrix. If  $|\text{adj} (24A)| = \text{adj}(3\text{adj}(2A))$ , then  $|A|^2$  is equal to :  
 (A)  $6^6$       (B)  $2^{12}$       (C)  $2^6$       (D) 1

**Official Ans. by NTA (C)**

**Allen Ans. (C)**

**Sol.**  $|\text{adj} (24A)| = |\text{adj} 3(\text{adj} 2A)|$   
 $\Rightarrow |24A|^2 = (3 \text{adj}(2A))^2$   
 $\Rightarrow (24^3 |A|^2)^2 = (3^3 |\text{adj} (2A)|)^2$   
 $= 3^6 (|2A|^2)^2$   
 $\Rightarrow 24^6 |A|^2 = (24^3 |A|)^2 = 3^6 \times 2^{12} |A|^4$   
 $\Rightarrow |A|^2 = \frac{24^6}{3^6 \times 2^{12}} = 64$

4. The ordered pair (a, b), for which the system of linear equations

$$3x - 2y + z = b$$

$$5x - 8y + 9z = 3$$

$$2x + y + az = -1$$

has no solution, is :

- (A)  $\left(3, \frac{1}{3}\right)$       (B)  $\left(-3, \frac{1}{3}\right)$   
 (C)  $\left(-3, -\frac{1}{3}\right)$       (D)  $\left(3, -\frac{1}{3}\right)$

**Official Ans. by NTA (C)**

**Allen Ans. (C)**

**Sol.** 
$$\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0$$
  
 $3(-8a - 9) + 2(5a - 18) + 1(21) = 0$

$$\Rightarrow a = -3$$

Also  $\Delta_2 = \begin{vmatrix} 3 & -2 & b \\ 5 & 8 & 3 \\ 2 & 1 & -1 \end{vmatrix}^{\frac{1}{3}}$

If  $b = \frac{1}{3}$

$$\Delta_2 = 0$$

So b must be equal to

$$-\frac{1}{3}$$

5. The remainder when  $(2021)^{2023}$  is divided by 7 is :  
 (A) 1      (B) 2      (C) 5      (D) 6

**Official Ans. by NTA (C)**

**Allen Ans. (C)**

**Sol.**  $(2021)^{2023} = (7\lambda - 2)^{2023}$   
 $= 2023C_0(7A)^{2023} - \dots - 2023C_{2023}2^{2023}$   
 $= 7t - 2^{2023}$

$$\therefore -2^{2023} = -2 \times 2^{2022}$$

$$= -2 \times (2^3)^{674}$$

$$= -2(1 + 7\mu)^{674}$$

$$= -(7\alpha + 2)$$

$$\Rightarrow \text{remainder} = -2 \text{ or } +5$$

6.  $\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$  is equal to :  
 (A)  $\sqrt{2}$       (B)  $-\sqrt{2}$   
 (C)  $\frac{1}{\sqrt{2}}$       (D)  $-\frac{1}{\sqrt{2}}$

**Official Ans. by NTA (D)**

**Allen Ans. (D)**

**Sol.**  $\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$

$$\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{\sin(\sin^{-1} \sqrt{1-x^2}) - x}{1 - \tan\left(\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right)}$$

$$\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{\sqrt{1-x^2} - x}{1 - \left(\frac{\sqrt{1-x^2}}{x}\right)}$$

$$\lim_{x \rightarrow \frac{1}{\sqrt{2}}} (-x) = -\frac{1}{\sqrt{2}}$$

7. Let  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  be two real valued functions

$$\text{defined as } f(x) = \begin{cases} -|x+3| & , \quad x < 0 \\ e^x & , \quad x \geq 0 \end{cases} \text{ and}$$

$$g(x) = \begin{cases} x^2 + k_1 x & , \quad x < 0 \\ 4x + k_2 & , \quad x \geq 0 \end{cases}, \text{ where } k_1 \text{ and } k_2 \text{ are}$$

real constants. If  $(gof)$  is differentiable at  $x = 0$ , then  $(gof)(-4) + (gof)(4)$  is equal to :

- (A)  $4(e^4 + 1)$       (B)  $2(2e^4 + 1)$   
 (C)  $4e^4$       (D)  $2(2e^4 - 1)$

**Official Ans. by NTA (D)**

**Allen Ans. (D)**

$$\text{Sol. } f(x) = \begin{cases} x+3 & ; \quad x < -3 \\ -(x+3) & ; \quad -3 \leq x < 0 \\ e^x & ; \quad x \geq 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 + k_1 x & ; \quad x < 0 \\ 4x + k_2 & ; \quad x \geq 0 \end{cases}$$

$$g(f(x)) = \begin{cases} f(x)^2 + k_1 f(x) & ; \quad f(x) < 0 \\ 4f(x) + k_2 & ; \quad f(x) \geq 0 \end{cases}$$

$$g(f(x)) = \begin{cases} (x+3)^2 + k_1(x+3) & ; \quad x < -3 \\ (x+3)^2 - k_1(x+3) & ; \quad -3 \leq x < 0 \\ 4e^x + k_2 & ; \quad x > 0 \end{cases}$$

check continuity at  $x = 0$

$$gof(0) = g(f(0^-)) = g(f(0^+))$$

$$4 + k_2 = 9 - 3k_1 = 4 + k_2$$

$$3k_1 + k_2 = 5 \quad \dots(a)$$

differentiate

$$(g(f(x)))' = \begin{cases} 2(x+3) + k_1 & ; \quad x < -3 \\ 2(x+3) - k_1 & ; \quad -3 \leq x < 0 \\ 4e^x & ; \quad x \geq 0 \end{cases}$$

$$6 - k_1 = 4$$

$$k_1 = 2 \quad \dots(b)$$

$$\therefore k_1 = 2, k_2 = -1$$

$$\text{gof}(x) = \begin{cases} (x+3)^2 + 2(x+3) & ; \quad x < -3 \\ (x+3)^2 - 2(x+3) & ; \quad -3 \leq x < 0 \\ 4e^x - 1 & ; \quad x \geq 0 \end{cases}$$

$$\text{gof}(-4) + \text{gof}(4) = 4e^4 - 2$$

$$\Rightarrow 2(2e^4 - 1)$$

8. The sum of the absolute minimum and the absolute maximum values of the function  $f(x) = |3x - x^2 + 2| - x$  in the interval  $[-1, 2]$  is :

$$(A) \frac{\sqrt{17} + 3}{2} \quad (B) \frac{\sqrt{17} + 5}{2}$$

$$(C) 5 \quad (D) \frac{9 - \sqrt{17}}{2}$$

**Official Ans. by NTA (A)**

**Allen Ans. (A)**

$$\text{Sol. } f(x) = \begin{cases} x^2 - 4x - 2, & \forall x \in \left(-1, \frac{3 - \sqrt{17}}{2}\right) \\ -x^2 + 2x + 2, & \forall x \in \left(\frac{3 - \sqrt{17}}{2}, 2\right) \end{cases}$$

$$f'(x) \text{ when } x \in \left(-1, \frac{3 - \sqrt{17}}{2}\right)$$

$$f'(x) = 2x - 4 = 0 \Rightarrow x = 2$$

$$f'(x) = 2(x - 2) \Rightarrow f'(x) \text{ is always } \downarrow$$

$$f(2) = 2$$

$$f(-1) = 3$$

$$f\left(\frac{3 - \sqrt{17}}{2}\right) = \frac{\sqrt{17} - 3}{2}$$

$$f'(x) \text{ when } x \in \left(\frac{3 - \sqrt{17}}{2}, 2\right)$$

$$f'(x) = -2x + 2$$

$$f'(x) = -2(x - 1)$$

$$f'(x) = 0 \text{ when } x = 1$$

$$f(1) = 3$$

$$\text{absolute minimum value} = \frac{\sqrt{17} - 3}{2}$$

$$\text{absolute maximum value} = 3$$

$$\text{Sum} = \frac{\sqrt{17} - 3}{2} + 3 = \frac{\sqrt{17} + 3}{2}$$

9. Let S be the set of all the natural numbers, for which the line  $\frac{x}{a} + \frac{y}{b} = 2$  is a tangent to the curve

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2 \text{ at the point } (a, b), ab \neq 0. \text{ Then:}$$

- (A)  $S = \emptyset$  (B)  $n(S) = 1$   
 (C)  $S = \{2k : k \in \mathbb{N}\}$  (D)  $S = \mathbb{N}$

**Official Ans. by NTA (D)**

**Allen Ans. (D)**

**Sol.**  $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$

Slope of tangent at  $(a, b)$

$$n\left(\frac{x}{a}\right)^{n-1} \cdot \frac{1}{a} + n\left(\frac{y}{b}\right)^{n-1} \cdot \frac{1}{b} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} \Big|_{(a,b)} = -\frac{b}{a}$$

$\therefore$  Equation of tangent

$$y - b = -\frac{b}{a} (x - a)$$

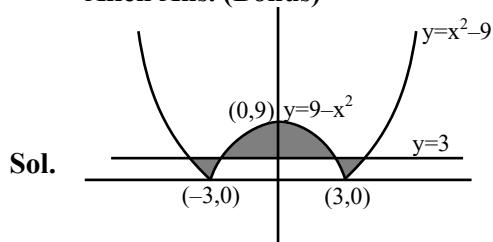
$$\frac{x}{a} + \frac{y}{b} = 2 \quad \forall n \in \mathbb{N}$$

10. The area bounded by the curve  $y = |x^2 - 9|$  and the line  $y = 3$  is :

- (A)  $4(2\sqrt{3} + \sqrt{6} - 4)$  (B)  $4(4\sqrt{3} + \sqrt{6} - 4)$   
 (C)  $8(4\sqrt{3} + 3\sqrt{6} - 9)$  (D)  $8(4\sqrt{3} + \sqrt{6} - 9)$

**Official Ans. by NTA (D)**

**Allen Ans. (Bonus)**



Area of shaded region

$$\begin{aligned} &= 2 \int_0^3 \left( \sqrt{9+y} - \sqrt{9-y} \right) dy + 2 \int_3^9 \left( \sqrt{9-y} \right) dy \\ &= 2 \left[ \int_0^3 (9+y)^{1/2} dy - \int_0^3 (9-y)^{1/2} dy + \int_3^9 (9-y)^{1/2} dy \right] \\ &= 2 \left[ \frac{2}{3} \left[ (9+y)^{3/2} \right]_0^3 + \frac{2}{3} \left[ (9-y)^{3/2} \right]_0^3 - \frac{2}{3} \left[ (9-y)^{3/2} \right]_3^9 \right] \\ &= \frac{4}{3} \left[ 12\sqrt{12} - 27 + 6\sqrt{6} - 27 - (0 - 6\sqrt{6}) \right] \\ &= \frac{4}{3} [24\sqrt{3} + 12\sqrt{6} - 54] \\ &= 8(4\sqrt{3} + 2\sqrt{6} - 9) \end{aligned}$$

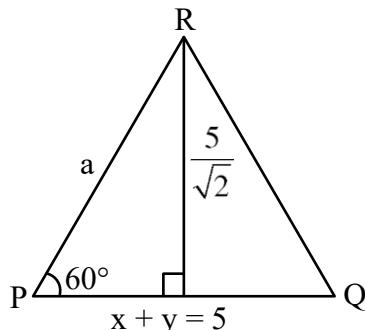
11. Let R be the point  $(3, 7)$  and let P and Q be two points on the line  $x + y = 5$  such that PQR is an equilateral triangle. Then the area of  $\Delta PQR$  is :

- (A)  $\frac{25}{4\sqrt{3}}$  (B)  $\frac{25\sqrt{3}}{2}$  (C)  $\frac{25}{\sqrt{3}}$  (D)  $\frac{25}{2\sqrt{3}}$

**Official Ans. by NTA (D)**

**Allen Ans. (D)**

**Sol.**



$$\sin 60^\circ = \frac{5/\sqrt{2}}{a}$$

$$a = \frac{5\sqrt{2}}{3}$$

$$\text{Area of } \Delta PQR = \frac{\sqrt{3}}{4} a^2 = \frac{25}{2\sqrt{3}}$$

12. Let C be a circle passing through the points  $A(2, -1)$  and  $B(3, 4)$ . The line segment AB is not a diameter of C. If r is the radius of C and its centre

lies on the circle  $(x - 5)^2 + (y - 1)^2 = \frac{13}{2}$ , then  $r^2$  is

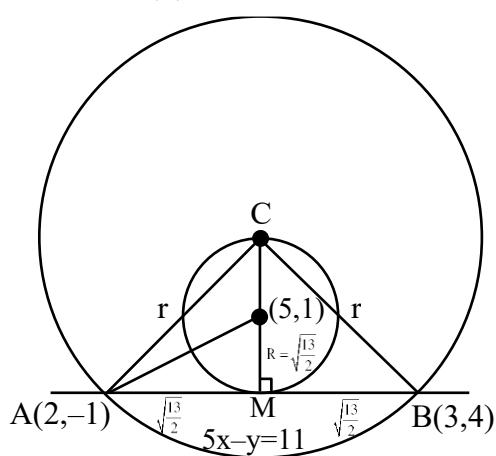
equal to :

- (A) 32 (B)  $\frac{65}{2}$  (C)  $\frac{61}{2}$  (D) 30

**Official Ans. by NTA (B)**

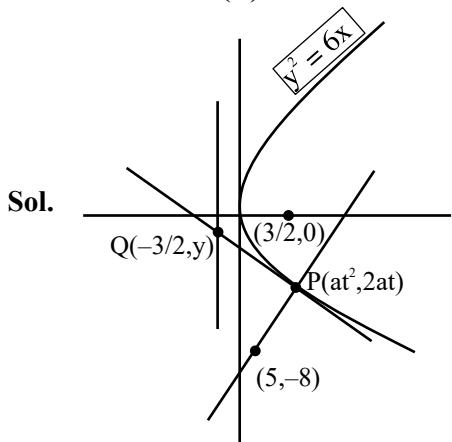
**Allen Ans. (B)**

**Sol.**



$$\begin{aligned} AB &= \sqrt{26} \\ r^2 &= CM^2 + AM^2 \\ &= \left(2 \times \sqrt{\frac{13}{2}}\right)^2 + \left(\sqrt{\frac{13}{2}}\right)^2 \\ r^2 &= \frac{65}{2} \end{aligned}$$

13. Let the normal at the point P on the parabola  $y^2 = 6x$  pass through the point  $(5, -8)$ . If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is :
- (A) -3      (B)  $-\frac{9}{4}$       (C)  $-\frac{5}{2}$       (D) -2

**Official Ans. by NTA (B)****Allen Ans. (B)**

$$\text{Equation of normal : } y = -tx + 2at + at^3 \quad \left(a = \frac{3}{2}\right)$$

since passing through  $(5, -8)$ , we get  $t = -2$

Co-ordinate of Q :  $(6, -6)$

Equation of tangent at Q :  $x + 2y + 6 = 0$

$$\text{Put } x = \frac{-3}{2} \text{ to get } R\left(\frac{-3}{2}, \frac{-9}{4}\right)$$

14. If the two lines  $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = z-2$  and  $l_2 : \frac{x-1}{1} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$  are perpendicular, then an angle between the lines  $l_2$  and  $l_3 : \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$  is :

- (A)  $\cos^{-1}\left(\frac{29}{4}\right)$       (B)  $\sec^{-1}\left(\frac{29}{4}\right)$   
 (C)  $\cos^{-1}\left(\frac{2}{29}\right)$       (D)  $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

**Official Ans. by NTA (B)****Allen Ans. (B)**

$$\text{Sol. } l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$$

$$l_2 : \frac{x-1}{1} = \frac{y+3/2}{\alpha/2} = \frac{z+5}{2}$$

$$l_3 : \frac{x-1}{-3} = \frac{y-1/2}{-2} = \frac{z-0}{4}$$

$$l_1 \perp l_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}+4}} = 0 \Rightarrow \alpha = 3$$

angle between  $l_2$  &  $l_3$

$$\cos \theta = \frac{|1 \times (-3) + (-2)(\alpha/2) + 2 \times 4|}{\sqrt{1+4+\frac{\alpha^2}{4}}\sqrt{9+16+4}}$$

$$\cos \theta = \frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$$

put  $\alpha = 3$

$$\cos \theta = \frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}} = \frac{4}{29}$$

$$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$$

15. Let the plane  $2x + 3y + z + 20 = 0$  be rotated through a right angle about its line of intersection with the plane  $x - 3y + 5z = 8$ . If the mirror image of the point  $\left(2, -\frac{1}{2}, 2\right)$  in the rotated plane is  $B(a, b, c)$ , then :

- (A)  $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$       (B)  $\frac{a}{4} = \frac{b}{5} = \frac{c}{-2}$   
 (C)  $\frac{a}{8} = \frac{b}{-5} = \frac{c}{4}$       (D)  $\frac{a}{4} = \frac{b}{5} = \frac{c}{2}$

**Official Ans. by NTA (A)****Allen Ans. (A)**



**Sol.**  $\sigma^2 = \frac{\sum_{i=1}^5 (x_i - \bar{x})^2}{n}$

Mean = 6

$$\frac{a+b+8+5+10}{5} = 6$$

$$a+b = 7$$

$$b = 7 - a$$

$$6.8 = \frac{(a-6)^2 + (b-6)^2 + (8-6)^2 + (5-6)^2 + (10-6)^2}{5}$$

$$34 = (a-6)^2 + (7-a-6)^2 + 4 + 1 + 18$$

$$a^2 - 7a + 12 = 0 \Rightarrow a = 4 \text{ or } a = 3$$

$$a = 4 \quad a = 3$$

$$b = 3 \quad b = 4$$

$$M = \frac{\sum_{i=1}^5 |x_i - \bar{x}|}{n}$$

$$M = \frac{|a-6| + |b-6| + |8-6| + |5-6| + |10-6|}{5}$$

$$\text{when } a = 3, b = 4$$

$$M = \frac{3+2+2+1+4}{5}$$

$$M = \frac{12}{5}$$

$$25M = 25 \times \frac{12}{5} = 60$$

- 19.** Let  $f(x) = 2\cos^{-1}x + 4\cot^{-1}x - 3x^2 - 2x + 10$ ,  $x \in [-1, 1]$ . If  $[a, b]$  is the range of the function then  $4a - b$  is equal to:

(A) 11      (B)  $11 - \pi$     (C)  $11 + \pi$     (D)  $15 - \pi$

**Official Ans. by NTA (B)**

**Allen Ans. (B)**

**Sol.**  $f'(x) = \frac{-2}{\sqrt{1-x^2}} - \frac{4}{1+x^2} - 6x - 2$

$$= -2 \left[ \frac{1}{\sqrt{1-x^2}} + \frac{2}{1+x^2} + 3x + 1 \right]$$

$f'(x) < 0 \Rightarrow f(x)$  is a dec. function

$$f(1) = \pi + 5$$

$$f(-1) = 5\pi + 5$$

Range :  $[a, b] \equiv [\pi + 5, 5\pi + 5]$

$$a = \pi + 5, b = 5\pi + 5 \Rightarrow 4a - b = 11 - \pi.$$

**20.** Let  $\Delta, \nabla \in \{\wedge, \vee\}$  be such that

$p \nabla q \Rightarrow ((p \Delta q) \nabla r)$  is a tautology.

Then  $(p \nabla q) \Delta r$  is logically equivalent to :

(A)  $(p \Delta r) \vee q$                 (B)  $(p \Delta r) \wedge q$

(C)  $(p \wedge r) \Delta q$                 (D)  $(p \nabla r) \wedge q$

**Official Ans. by NTA (A)**

**Allen Ans. (A)**

**Sol.** **Case-I** If  $\Delta \equiv \nabla \equiv \wedge$

$$(p \wedge q) \rightarrow ((p \wedge q) \wedge r)$$

it can be false if  $r$  is false,

so not a tautology

**Case-II** If  $\Delta \equiv \nabla \equiv \vee$

$$(p \vee q) \rightarrow ((p \vee q) \vee r) \equiv \text{tautology}$$

then  $(p \vee q) \vee r \equiv (p \Delta r) \vee q$

**Case-III** if  $\Delta = \vee, \nabla = \wedge$

$$(p \wedge q) \rightarrow \{(p \vee q) \wedge r\}$$

Not a tautology

(Check  $p \rightarrow T, q \rightarrow T, r \rightarrow F$ )

**Case-IV** if  $\Delta = \wedge, \nabla = \vee$

$$(p \wedge q) \rightarrow \{(p \wedge q) \vee r\}$$

Not a tautology

## SECTION-B

- 1.** The sum of the cubes of all the roots of the equation  $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$  is \_\_\_\_\_.

**Official Ans. by NTA (36)**

**Allen Ans. (36)**

**Sol.**  $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$

$x = 0$  is not the root of this equation so divide it by  $x^2$

$$x^2 - 3x - 2 + \frac{3}{x} + \frac{1}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 2 + 2 - 3\left(x - \frac{1}{x}\right) - 2 = 0$$

$$\left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right) = 0$$

$$x - \frac{1}{x} = 0,$$

$$x - \frac{1}{x} = 3$$

$$x^2 - 1 = 0$$

$$x^2 - 3x - 1 = 0$$

$$x = \pm 1$$

$$\gamma + \delta = 3$$

$$\alpha = 1, \beta = -1$$

$$\gamma\delta = -1$$

$$\alpha^3 + \beta^3 + \gamma^3 + \delta^3$$

$$1 - 1 + (\gamma + \delta)((\gamma + \delta)^2 - 3\gamma\delta)$$

$$0 + 3(9 - 3(-1))$$

$$+ 3(12) = 36$$

- 2.** There are ten boys  $B_1, B_2, \dots, B_{10}$  and five girls  $G_1, G_2, \dots, G_5$  in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both  $B_1$  and  $B_2$  together should not be the members of a group, is \_\_\_\_\_.

**Official Ans. by NTA (1120)**

**Allen Ans. (1120)**

**Sol.**  $n(B) = 10$

$n(a) = 5$

The number of ways of forming a group of 3 girls of 3 boys.

$$= {}^{10}C_3 \times {}^5C_3$$

$$= \frac{10 \times 9 \times 8}{3 \times 2} \times \frac{5 \times 4}{2} = 1200$$

The number of ways when two particular boys  $B_1$  of  $B_2$  be the member of group together

$$= {}^8C_1 \times {}^5C_3 = 8 \times 10 = 80$$

Number of ways when boys  $B_1$  of  $B_2$  hot in the same group together

$$= 1200 \times 80 = 1120$$

- 3.** Let the common tangents to the curves  $4(x^2 + y^2) = 9$  and  $y^2 = 4x$  intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and l respectively denote the eccentricity and the length of the latus rectum of this ellipse, then  $\frac{l}{e^2}$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (4)**

**Allen Ans. (4)**

**Sol.**  $x^2 + y^2 = \frac{9}{4}$   $y = 4x$

Equation tangent in slope form

$$y = mx \pm \frac{3}{2} \sqrt{(1+m^2)} \quad \dots(1)$$

$$y = mx + \frac{1}{m} \quad \dots(2)$$

compare (1) & (2)

$$\pm \frac{3}{2} \sqrt{(1+m^2)} = \frac{1}{m^2}$$

$$9m^2(1+m^2) = 4$$

$$9m^4 + 9m^2 - 4 = 0$$

$$9m^4 + 12m^2 - 3m^2 - 4 = 0$$

$$3m^2(3m^2 + 4) - (3m^2 + 4) = 0$$

$$m^2 = -\frac{4}{3} \text{ (Rejected)}$$

$$m^2 = \frac{1}{3} \Rightarrow m = \pm \frac{1}{\sqrt{3}}$$

Equation of common tangent

$$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$$

$$\text{on X axis } y = 0$$

$$OQ = -3$$

$$b = |OQ| = 3$$

$$a = 6$$

$$b^2 = a^2(1 - e^2) \Rightarrow e^2 = 1 - \frac{9}{36} = \frac{3}{4}$$

$$e = \frac{2b^2}{a} = \frac{2 \times 9}{6} = 3$$

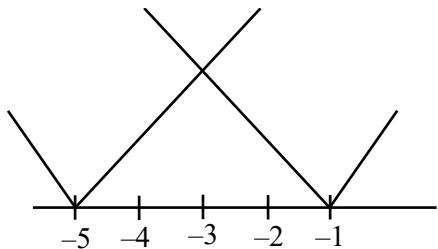
$$\frac{e}{e^2} = \frac{3}{3/4} = 4$$

4. Let  $f(x) = \max\{|x+1|, |x+2|, \dots, |x+5|\}$ . Then  $\int_{-6}^0 f(x) dx$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (21)**

**Allen Ans. (21)**

**Sol.**  $f(x) = \max\{|x+1|, |x+2|, |x+3|, |x+4|, |x+5|\}$



$$\begin{aligned} \int_{-6}^0 f(x) dx &= \int_{-6}^{-3} |x+1| dx + \int_{-3}^0 |x+5| dx \\ &= - \int_{-6}^{-3} (x+1) dx + \int_{-3}^0 (x+5) dx \\ &= - \left[ \frac{x^2}{2} + x \right]_{-6}^{-3} + \left[ \frac{x^2}{2} + 5x \right]_{-3}^0 \\ &= - \left[ \left( \frac{9}{2} - 3 \right) - (18 - 6) \right] + \left[ 0 - \left( \frac{9}{2} - 15 \right) \right] \\ &= - \left[ \frac{3}{2} - 12 \right] + \frac{21}{2} = \frac{21}{2} + \frac{21}{2} = 21 \end{aligned}$$

5. Let the solution curve  $y = y(x)$  of the differential equation  $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$  pass through the origin. Then  $y(2)$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (12)**

**Allen Ans. (12)**

**Sol.**  $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx$

$$(x^2 + 4) \frac{dy}{dx} = 2x^3 + 6xy + 8x$$

$$(x^2 + 4) \frac{dy}{dx} - 6xy = 2x^3 + 8x$$

$$\frac{dy}{dx} - \frac{6x}{x^2 + 4}y = \frac{2x^3 + 8x}{x^2 + 4}$$

$$\text{L.I. } \frac{dy}{dx} + py = \phi$$

$$\begin{aligned} p &= \frac{-6x}{x^2 + 4} & \phi &= \frac{2x^3 + 8x}{x^2 + 4} \\ \text{I.F.} &= e^{-\int \frac{6x}{x^2 + 4} dx} = e^{-3\log_e(x^2 + 4)} \\ &= e^{\log_e(x^2 + 4)^{-3}} = \frac{1}{(x^2 + 4)^3} \end{aligned}$$

**Sol.**

$$y \cdot \frac{1}{(x^2 + 4)^3} = \int \frac{2x^3 + 8x}{(x^2 + 4)^3(x^2 + 4)} dx$$

$$\frac{y}{(x^2 + 4)^3} = \int \frac{2x(x^2 + 4)}{(x^2 + 4)^3(x^2 + 4)} dx$$

$$x^2 + 4 = t$$

$$2xdx = dt$$

$$\frac{y}{(x^2 + 4)^3} = \int \frac{dt}{t^3}$$

$$\frac{y}{(x^2 + 4)^3} = \frac{-1}{2(x^2 + 4)^2} + C$$

passes through origin  $(0, 0)$

$$0 = \frac{-1}{2 \times 16} + C$$

$$\frac{y}{(x^2 + 4)^3} = \frac{-1}{2(x^2 + 4)^2} + \frac{1}{32}$$

$$y = \frac{-(x^2 + 4)}{2} + \frac{(x^2 + 4)^3}{32}$$

$$y(2) = -\frac{8}{2} + \frac{8 \times 8 \times 8}{32} = 12$$

6. If  $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ) = \alpha - \frac{1}{16} \sin(10^\circ)$ , then  $16 + \alpha^{-1}$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (80)**

**Allen Ans. (80)**

**Sol.**  $\sin 10^\circ \left( \frac{1}{2} \cdot 2 \sin 20^\circ \sin 40^\circ \right) \cdot \sin 10^\circ \sin(60^\circ - 10^\circ) \sin(60^\circ + 10^\circ)$

$$\sin 10^\circ \frac{1}{2} (\cos 20^\circ - \cos 60^\circ) \cdot \frac{1}{4} \sin 30^\circ$$

$$\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \sin 10^\circ \left( \cos 20^\circ - \frac{1}{2} \right)$$

$$\begin{aligned}
 &= \frac{1}{32} (2 \sin 10^\circ \cos 20^\circ - \sin 10^\circ) \\
 &= \frac{1}{32} (\sin 30^\circ - \sin 10^\circ - \sin 10^\circ) \\
 &= \frac{1}{32} \left( \frac{1}{2} - 2 \sin 10^\circ \right) \\
 &= \frac{1}{64} (1 - 4 \sin 10^\circ) \\
 &= \frac{1}{64} - \frac{1}{16} \sin 10^\circ
 \end{aligned}$$

Hence  $\alpha = \frac{1}{64}$

$$16 + \alpha^{-1} = 80$$

7. Let  $A = \{n \in N : H.C.F. (n, 45) = 1\}$  and  
Let  $B = \{2k : k \in \{1, 2, \dots, 100\}\}$ . Then the sum of all the elements of  $A \cap B$  is \_\_\_\_\_.

**Official Ans. by NTA (5264)**

**Allen Ans. (5264)**

**Sol.** Sum of elements in  $A \cap B$

$$\begin{aligned}
 &= \underbrace{(2+4+6+\dots+200)}_{\text{Multiple of 2}} - \underbrace{(6+12+...+198)}_{\text{Multiple of 2 & 3 i.e. 6}} \\
 &\quad - \underbrace{(10+20+\dots+200)}_{\text{Multiple of 5 & 2 i.e. 10}} + \underbrace{(30+60+\dots+180)}_{\text{Multiple of 2, 5 & 3 i.e. 30}} \\
 &= 5264
 \end{aligned}$$

8. The value of the integral  $\frac{48}{\pi^4} \int_0^\pi \left( \frac{3\pi x^2}{2} - x^3 \right) \frac{\sin x}{1 + \cos^2 x} dx$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (6)**

**Allen Ans. (6)**

**Sol.**  $I = \frac{48}{\pi^4} \int_0^\pi x^2 \left( \frac{3\pi}{2} - x \right) \frac{\sin x}{1 + \cos^2 x} dx \dots (1)$

Apply king property

$$I = \frac{48}{\pi^4} \int_0^\pi (\pi - x)^2 \left( \frac{\pi}{2} + x \right) \frac{\sin x}{1 + \cos^2 x} dx \dots (2)$$

$$(1) + (2)$$

$$I = \frac{12}{\pi^3} \int_0^\pi \frac{\sin x}{1 + \cos^2 x} [\pi^2 + (\pi - 2)x(\pi - 2x)] dx \dots (3)$$

Apply king again

$$I = \frac{12}{\pi^3} \int_0^\pi \frac{\sin x}{1 + \cos^2 x} [\pi^2 + (\pi - 2)(\pi - x)(2x - \pi)] dx \dots (4)$$

$$(3) + (4)$$

$$I = \frac{6}{\pi^2} \int_0^\pi \frac{\sin x}{1 + \cos^2 x} [2\pi + (\pi - 2)(\pi - 2x)] dx \dots (5)$$

Apply king

$$I = \frac{6}{\pi^2} \int_0^\pi \frac{\sin x}{1 + \cos^2 x} [2\pi + (\pi - 2)(2x - \pi)] dx \dots (6)$$

$$(5) + (6)$$

$$I = \frac{12}{\pi} \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx$$

$$\text{Let } \cos x = t \Rightarrow \sin x dx = -dt$$

$$I = \frac{12}{\pi} \int_1^{-1} \frac{-dt}{1+t^2} = 6$$

9. Let  $A = \sum_{i=1}^{10} \sum_{j=1}^{10} \min\{i, j\}$  and

$$B = \sum_{i=1}^{10} \sum_{j=1}^{10} \max\{i, j\}. \text{ Then } A + B \text{ is equal to }$$

\_\_\_\_\_.

**Official Ans. by NTA (1100)**

**Allen Ans. (1100)**

**Sol.**  $A = \sum_{i=1}^{10} \sum_{j=1}^{10} \min\{i, j\}$

$$B = \sum_{i=1}^{10} \sum_{j=1}^{10} \max\{i, j\}$$

$$A = \sum_{j=1}^{10} \min(i, 1) + \min(j, 2) + \dots + \min(i, 10)$$

$$= \underbrace{(1+1+1+\dots+1)}_{19 \text{ times}} + \underbrace{(2+2+2\dots+2)}_{17 \text{ times}} + \underbrace{(3+3+3\dots+3)}_{15 \text{ times}}$$

$$+ \dots (1) \text{ 1 times}$$

$$B = \sum_{j=1}^{10} \max(i, 1) + \max(j, 2) + \dots + \max(i, 10)$$

$$= \underbrace{(10+10+\dots+10)}_{19 \text{ times}} + \underbrace{(9+9+\dots+9)}_{17 \text{ times}} + \dots + 1 \text{ 1 times}$$

$$\begin{aligned} A + B &= 20(1 + 2 + 3 + \dots + 10) \\ &= 20 \times \frac{10 \times 11}{2} = 10 \times 110 = 1100 \end{aligned}$$

10. Let  $S = (0, 2\pi) - \left\{ \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4} \right\}$ . Let  $y = y(x)$ ,  $x \in S$ , be the solution curve of the differential equation  $\frac{dy}{dx} = \frac{1}{1 + \sin 2x}$ ,  $y\left(\frac{\pi}{4}\right) = \frac{1}{2}$ . if the sum of abscissas of all the points of intersection of the curve  $y = y(x)$  with the curve  $y = \sqrt{2} \sin x$  is  $\frac{k\pi}{12}$ , then  $k$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (42)**

**Allen Ans. (42)**

**Sol.**  $\frac{dy}{dx} = \frac{1}{1 + \sin 2x}$

$$\int dy = \int \frac{dx}{(\sin x + \cos x)^2}$$

$$\int dy = \int \frac{\sec^2 x}{(1 + \tan x)^2}$$

$$y(x) = -\frac{1}{1 + \tan x} + C$$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{2} = -\frac{1}{2} + C$$

$$C = 1$$

$$y(x) = \frac{-1}{1 + \tan x} + 1$$

$$y(x) = \frac{-1 + 1 + \tan x}{1 + \tan x}$$

$$y(x) = \frac{\tan x}{1 + \tan x}$$

Solving with  $y = \sqrt{2} \sin x$

$$\frac{\tan x}{1 + \tan x} = \sqrt{2} \sin x$$

$$\sin x = 0, \quad \frac{1}{\sqrt{2}} = \sin x + \cos x$$

$$x = \pi \quad \frac{1}{2} = \sin\left(x + \frac{\pi}{4}\right)$$

$$\sin \frac{\pi}{6} = \sin\left(x + \frac{\pi}{4}\right)$$

$$x + \frac{\pi}{4} = \pi - \frac{\pi}{6}, \quad 2\pi + \frac{\pi}{6}$$

$$x = \frac{5\pi}{6} - \frac{\pi}{4}, \quad x = \frac{13\pi}{6} - \frac{\pi}{4}$$

$$x = \frac{7\pi}{12}, \quad x = \frac{23\pi}{12}$$

sum of sol.

$$= \pi + \frac{7\pi}{12} + \frac{23\pi}{12}$$

$$= \frac{12\pi + 7\pi + 23}{12} = \frac{42\pi}{12} = \frac{k\pi}{12}$$

$$\Rightarrow k = 42$$