JEE-MAIN – JUNE, 2022

(Held On Tuesday 27th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

Mathematics

Test Pattern : JEE-MAIN Maximum Marks : 120

Topic Covered: FULL SYLLABUS

Important instruction:

1. Use Blue / Black Ball point pen only.
2. There are three sections of equal weightage in the question paper **Physics, Chemistry** and **Mathematics** having 30 questions in each subject. Each paper have 2 sections A and B.
3. You are awarded +4 marks for each correct answer and –1 marks for each incorrect answer.
4. Use of calculator and other electronic devices is not allowed during the exam.
5. No extra sheets will be provided for any kind of work.

Name of the Candidate (in Capitals) _____

Father's Name (in Capitals) _____

Form Number : in figures _____

: in words _____

Centre of Examination (in Capitals): _____

Candidate's Signature: _____ Invigilator's Signature : _____

Rough Space

YOUR TARGET IS TO SECURE GOOD RANK IN JEE-MAIN

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Monday 27th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. The number of points of intersection of $|z-(4+3i)|=2$ and $|z|+|z-4|=6$, $z \in C$ is :

(A) 0 (B) 1

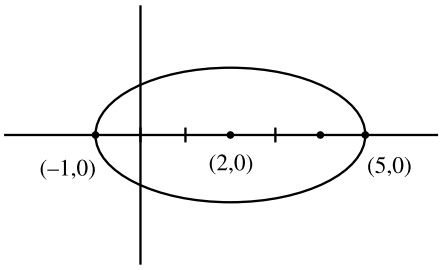
(C) 2 (D) 3

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $C : (x-4)^2 + (y-3)^2 = 4$

$E : \frac{(x-2)^2}{9} + \frac{y^2}{5} = 1$



Lower Extremity of vertical diameter of circle $\rightarrow (4,1)$

Put in ellipse $\Rightarrow \frac{(4-2)^2}{9} + \frac{1}{5} - 1$

$= \frac{4}{9} + \frac{1}{5} - 1$

$= \frac{29}{45} - 1 < 0$

Two Solutions

Answer (C)

2. Let $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $a \in R$. Then the sum of which the squares of all the values of a for $2f'(10) - f'(5) + 100 = 0$ is :

(A) 117 (B) 106

(C) 125 (D) 136

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$

$f(x) = a \begin{vmatrix} 1 & -1 & 0 \\ x & a & -1 \\ x^2 & ax & a \end{vmatrix}$

$= a\,[1\,(a^2 + ax) + 1\,(ax + x^2)]$

$\Rightarrow f(x) = a\,(x + a)^2$

so, $f'(x) = 2a\,(x + a)$

as, $2f'(10) - f'(5) + 100 = 0$

$\Rightarrow 2 \times 2a\,(10 + a) - 2a\,(5 + a) + 100 = 0$

$\Rightarrow 40a + 4a^2 - 10a - 2a^2 + 100 = 0$

$2a^2 + 30a + 100 = 0$

$\Rightarrow a^2 + 15a + 50 = 0$

$(a + 10)(a + 5) = 0$

$a = -10$ or $a = -5$

Required $= (-10)^2 + (-5)^2 = 125$

3. Let for some real numbers α and β, $a = \alpha - i\beta$. If the system of equations $4ix + (1+i)y = 0$ and $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$ has more than one solution then $\frac{\alpha}{\beta}$ is equal to :

(A) $-2+\sqrt{3}$ (B) $2-\sqrt{3}$

(C) $2+\sqrt{3}$ (D) $-2-\sqrt{3}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $a = \alpha - i\beta$; $\alpha \in R$; $\beta \in R$

$4ix + (1 + i)\, y = 0$ and

$$8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$$

$$\begin{vmatrix} 4i & 1+i \\ 8e^{i2\pi/3} & \bar{a} \end{vmatrix} = 0$$

$\Rightarrow 4i\,\bar{a} - (1 + i)\; 8e^{i2\pi/3} = 0$

$\Rightarrow 4i\,(\alpha + i\beta) - 8\,(1 + i)\left(\dfrac{-1+i\sqrt{3}}{2}\right) = 0$

$\Rightarrow i\alpha - \beta + 1 + \sqrt{3} + i\,(1 - \sqrt{3}) = 0$

$\Rightarrow \beta = \sqrt{3} + 1$

$\alpha = \sqrt{3} - 1$

So, $\dfrac{\alpha}{\beta} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$

4. Let A and B be two 3 × 3 matrices such that $AB = I$ and $|A| = \frac{1}{8}$ then $|\text{adj}(B\,\text{adj}(2A))|$ is equal to

(A) 16 (B) 32

(C) 64 (D) 128

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $AB = i$

$|\text{adj}\,(B\,\text{adj}\,(2A)| = |B\,\text{adj}\,(2A)|^2$

$= |B|^2\,|\text{adj}\,(2A)|^2$

$= |B|^2\,(|2A|^2)^2 = |B|^2\,(2^6\,|A|^2)^2$

$|A| = \frac{1}{8}$ and $|AB| = 1 \Rightarrow |A|\,|B| = 1$

$\Rightarrow \frac{1}{8}\,|B| = 1$

$\Rightarrow |B| = 8$

required value = 64

5. Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + ...$ then 4S is equal to

(A) $\left(\frac{7}{3}\right)^2$ (B) $\frac{7^3}{3^2}$

(C) $\left(\frac{7}{3}\right)^3$ (D) $\frac{7^2}{3^3}$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + ...$

Considering infinite sequence,

$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + ...$

$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + ...$

$\Rightarrow \frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \frac{10}{7^4} + ...$

$\Rightarrow \frac{6S}{7^2} = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \frac{8}{7^4} + ...$

$\frac{6S}{7}\left(1 - \frac{1}{7}\right) = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + ...$

$\Rightarrow \frac{6^2 S}{7^2} = \frac{2}{1 - \frac{1}{7}} = \frac{2}{6} \times 7$

$\Rightarrow S = \frac{2 \times 7^3}{6^3} \Rightarrow 4S = \frac{7^3}{3^3} = \left(\frac{7}{3}\right)^3$

6. If $a_1, a_2, a_3 ....$ and $b_1, b_2, b_3 ....$ are A.P. and $a_1 = 2,\ a_{10} = 3,\ a_1 b_1 = 1 = a_{10} b_{10}$ then $a_4 b_4$ is equal to

(A) $\frac{35}{27}$ (B) 1

(C) $\frac{27}{28}$ (D) $\frac{28}{27}$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $a_1, a_2, a_3 ....$A.P. ; $a_1 = 2$; $a_{10} = 3$; $d_1 = \frac{1}{9}$

$b_1, b_2, b_3, ...$ A.P. ; $b_1 = \frac{1}{2}$; $b_{10} = \frac{1}{3}$; $d_2 = \frac{-1}{54}$

[Using $a_1 b_1 = 1 = a_{10} b_{10}$; d_1 & d_2 are common differences respectively]

$a_4 . b_4 = (2 + 3d_1)\left(\frac{1}{2} + 3d_2\right)$

$= \left(2 + \frac{1}{3}\right)\left(\frac{1}{2} - \frac{1}{18}\right)$

$= \left(\frac{7}{3}\right)\left(\frac{8}{18}\right) = \frac{28}{27}$

7. If m and n respectively are the number of local maximum and local minimum points of the function $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t}\,dt$, then the ordered pair (m, n) is equal to

(A) (3, 2) (B) (2, 3)

(C) (2, 2) (D) (3, 4)

Official Ans. by NTA (B)

Allen Ans. (B)

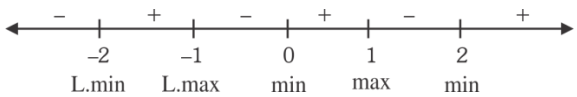
Sol. $m = L \cdot \max$

$N = L \cdot \min$

$$f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$$

$$f'(x) = \frac{(x^4 - 5x^2 + 4)2x}{2 + e^{x^2}} = \frac{2x(x^2 - 1)(x^2 - 4)}{2 + e^{x^2}}$$

$$= \frac{2x(x-1)(x+1)(x-2)(x+2)}{2 + e^{x^2}}$$



so, m = 2 and n = 3

8. Let f be a differentiable function in $\left(0, \frac{\pi}{2}\right)$. If $\int_{\cos x}^{1} t^2 f(t)\,dt = \sin^3 x + \cos x$ then $\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to :

(A) $6 - 9\sqrt{2}$ (B) $6 - \frac{9}{\sqrt{2}}$

(C) $\frac{9}{2} - 6\sqrt{2}$ (D) $\frac{9}{\sqrt{2}} - 6$

Official Ans. by NTA (B)

Allen Ans. (Bonus)

Sol. At right hand vicinity of x = 0 given equation does not satisfy

$$\because \text{LHS} = \int_{1^-}^{1} t^2 f(t)\,dt = 0,\ \text{RHS} = \lim_{x \to 0^+} \left(\sin^3 x + \cos x\right) = 1$$

LHS ≠ RHS hence data given in question is wrong hence BONUS

Correct data should have been

$$\int_{\cos x}^{1} t^2 f(t)\,dt = \sin^3 x + \cos x - 1$$

Calculation for option

differentiating both sides

$-\cos^2 x\, f(\cos x).(-\sin x) = 3\sin^2 x.\cos x - \sin x$

$\Rightarrow f(\cos x) = 3 \tan x - \sec^2 x$

$\Rightarrow f'(\cos x)(-\sin x) = 3\sec^2 x - 2\sec^2 x \tan x$

$\Rightarrow f'(\cos x)\cos x = \frac{2}{\cos^2 x} - \frac{3}{\sin x.\cos x}$

When $\cos x = \frac{1}{\sqrt{3}}$; $\sin x = \frac{\sqrt{2}}{\sqrt{3}}$

$$\therefore f'\left(\frac{1}{\sqrt{3}}\right)\frac{1}{\sqrt{3}} = 6 - \frac{9}{\sqrt{2}}.$$

9. The integral $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}} dx$, where [.] denotes the greatest integer function is equal to

(A) $1 + 6\log_e\left(\frac{6}{7}\right)$ (B) $1 - 6\log_e\left(\frac{6}{7}\right)$

(C) $\log_e\left(\frac{7}{6}\right)$ (D) $1 - 7\log_e\left(\frac{6}{7}\right)$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $$\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}} dx = -\int_1^0 \frac{1}{7^{\left[\frac{1}{x}\right]}} dx$$

$$= (-1)\left[\int_1^{1/2} \frac{1}{7} dx + \int_{1/2}^{1/3} \frac{1}{7^2} dx + \int_{1/3}^{1/4} \frac{1}{7^3} dx + \ldots\ldots\infty\right]$$

$$= \left(\frac{1}{7} + \frac{1}{2.7^2} + \frac{1}{3.7^3} + \ldots\infty\right) - \left(\frac{1}{7.2} + \frac{1}{7^2.3} + \frac{1}{7^2.4} \ldots\infty\right)$$

$$= -\ln\left(1 - \frac{1}{7}\right) - 7\left(\frac{1}{7^2.2} + \frac{1}{7^3.3} + \frac{1}{7^4.4} + \ldots\ldots\infty\right)$$

$$\left[\text{as } \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots\infty\right]$$

$$\left[\text{as } \ln(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} \ldots\infty\right)\right]$$

$$= -\ln\frac{6}{7} - 7\left(-\ln\left(1 - \frac{1}{7}\right) - \frac{1}{7}\right)$$

$$= 6\ln\frac{6}{7} + 1$$

10. If the solution curve of the differential equation $\left((\tan^{-1} y) - x\right)dy = (1 + y^2)dx$ passes through the point (1, 0) then the abscissa of the point on the curve whose ordinate is tan (1) is :

(A) 2e (B) $\frac{2}{e}$

(C) 2 (D) $\frac{1}{e}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $$\frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{\tan^{-1} y}{1 + y^2}$$

$$\text{I.f} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

$$xe^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1 + y^2} e^{\tan^{-1} y} dy$$

$x \cdot e^{\tan^{-1} y} = \left(\tan^{-1} y - 1\right) e^{\tan^{-1} y} + c$

$\because$ (1, 0) lies exit c = 2.

For y = tan1 $\Rightarrow x = \frac{2}{e}$

11. If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x + y - 29 = 0$, is $x^2 + ay^2 + bxy + cx + dy + k = 0$ then a + b + c +d + k is equal to

(A) 575 (B) –575

(C) 576 (D) –576

Official Ans. by NTA (D)

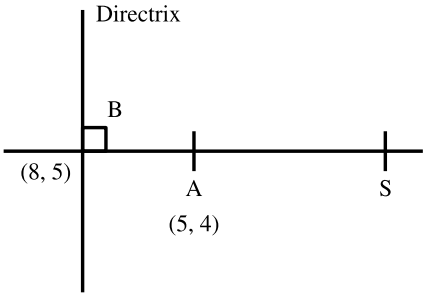
Allen Ans. (D)

Sol. Vertex (5,4)

Directrix : 3x + y – 29 = 0

Co-ordinates of B (foot of directrix)

$\frac{x-5}{3} = \frac{y-4}{1} = -\left(\frac{15+4-29}{10}\right) = 1$



x = 8, y = 5

S = (2, 3) (focus)

Equation of parabola

PS = PM

so equation is

$x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$

a + b + c + d + k = 9 – 6 + 134 – 2 – 711 = – 576

12. The set of values of k for which the circle C : $4x^2 + 4y^2 - 12x + 8y + k = 0$ lies inside the fourth quadrant and the point $\left(1, -\frac{1}{3}\right)$ lies on or inside the circle C is :

(A) An empty set (B) $\left(6, \frac{95}{9}\right]$

(C) $\left[\frac{80}{9}, 10\right)$ (D) $\left(9, \frac{92}{9}\right]$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. C : $4x^2 + 4y^2 - 12x + 8y + k = 0$

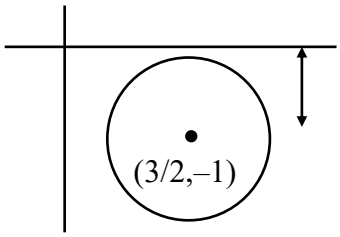
$\Rightarrow x^2 + y^2 - 3x + 2y + \left(\frac{k}{4}\right) = 0$

Centre $\left(\frac{3}{2}, -1\right)$; $r = \sqrt{\frac{13-k}{2}}$ $\Rightarrow k \le 13$ ... (1)

(i) Point $\left(1, \frac{-1}{3}\right)$ lies on or inside circle C

$\Rightarrow S_1 \le 0 \Rightarrow k \le \frac{92}{9}$ ... (2)

(ii) C lies in 4th quadrant



r < 1

$\Rightarrow \frac{\sqrt{13-k}}{2} < 1$

$\Rightarrow k < 9$ .... (3)

Hence $(1) \cap (2) \cap (3) \Rightarrow k \in \left(9, \frac{92}{9}\right]$

13. Let the foot of the perpendicular from the point (1, 2, 4) on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P. Then the distance of P from the plane $3x + 4y + 12z + 23 = 0$

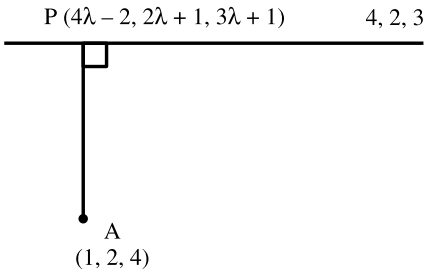
(A) 5 (B) $\frac{50}{13}$

(C) 4 (D) $\frac{63}{13}$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol.



$\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = \lambda$

(x, y, z) = (4λ – 2, 2λ + 1, 3λ – 1)

$\overrightarrow{AP} = (4\lambda - 3)\,\hat{i} + (2\lambda - 1)\,\hat{j} + (3\lambda - 5)\,\hat{k}$

$\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AP} \cdot \vec{b} = 0$

4(4λ – 3) + 2(2λ – 1) + 3(3λ – 5) = 0

29λ = 12 + 2 + 15 = 29

λ = 1

P = (2, 3, 2)

3x + 4y + 12z + 23 = 0

$$d = \left|\frac{6+12+24+23}{\sqrt{9+16+144}}\right|$$

$$d = \left|\frac{65}{13}\right| = 5$$

14. The shortest distance between the lines $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ and $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$ is :

(A) $\frac{18}{\sqrt{5}}$ (B) $\frac{22}{3\sqrt{5}}$

(C) $\frac{46}{3\sqrt{5}}$ (D) $6\sqrt{3}$

Official Ans. by NTA (A)
Allen Ans. (A)

Sol. $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$

$\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$

A = (3, 2, 1) B = (–3, 6, 5)

$\vec{n_1} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{n_2} = 2\hat{i} + \hat{j} - 3\hat{k}$

$\overrightarrow{BA} = 6\hat{i} - 4\hat{j} - 4\hat{k}$

SHORTEST DISTANCE = $\frac{\left[\overrightarrow{BA}\ \vec{n_1}\ \vec{n_2}\right]}{\left|\vec{n_1}\times\vec{n_2}\right|}$

$\vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix}$

$= 10\hat{i} - 8\hat{j} - 4\hat{k}$

$[\overrightarrow{BA}\ \vec{n_1}\ \vec{n_2}] = 60 + 32 + 16 = 108$

$|\vec{n_1} \times \vec{n_2}| = \sqrt{100+64+16} = \sqrt{180}$

S.D = $\frac{108}{\sqrt{180}} = \frac{108}{6\sqrt{5}} = \frac{18}{\sqrt{5}}$

15. Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonal of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute. $|\vec{a}|=1$ and $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$. If $\vec{c}=2\sqrt{2}(\vec{a}\times\vec{b})-2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is :

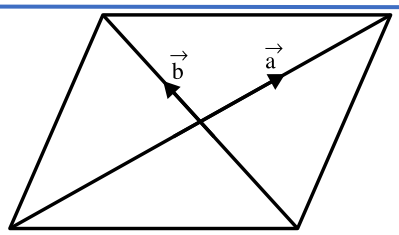
(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$

(C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{4}$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol.



Area = $\frac{1}{2}|\vec{a}\times\vec{b}| = 2\sqrt{2} \Rightarrow |\vec{a}\times\vec{b}| = 4\sqrt{2}$

$|\vec{a}| = 1$ and $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$

$\Rightarrow \cos\theta = \sin\theta$

$\Rightarrow \theta = \frac{\pi}{4}$

$\therefore |\vec{a}\times\vec{b}| = 4\sqrt{2} \Rightarrow |\vec{a}||\vec{b}|\sin\frac{\pi}{4} = 4\sqrt{2}$

$\Rightarrow |\vec{b}| = 8$

Now, $\vec{c} = 2\sqrt{2}(\vec{a}\times\vec{b}) - 2\vec{b}$

$|\vec{c}| = \sqrt{(2\sqrt{2})^2|\vec{a}\times\vec{b}|^2 + (2|\vec{b}|)^2} = 16\sqrt{2}$

Now, $\vec{b}.\vec{c} = -2|\vec{b}|^2$

$\Rightarrow 8\times16\sqrt{2}\times\cos\alpha = -2.64$

$\Rightarrow \cos\alpha = -\frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{3\pi}{4}$

16. The mean and variance of the data4, 5, 6, 6, 7, 8, x, y where x < y are 6, and $\frac{9}{4}$ respectively. Then $x^4 + y^2$ is equal to

(A) 162 (B) 320

(C) 674 (D) 420

Official Ans. by NTA (B)
Allen Ans. (B)

Sol. mean $\bar{x} = \frac{4+5+6+6+7+8+x+y}{8} = 6$

$\Rightarrow x + y = 48 - 36 = 12$

Variance

$= \frac{1}{8}(16+25+36+36+49+64+x^2+y^2) - 36 = \frac{9}{4}$

$\Rightarrow x^2 + y^2 = 80$

$\therefore x = 4; y = 8$

$x^4 + y^2 = 256 + 64 = 320$

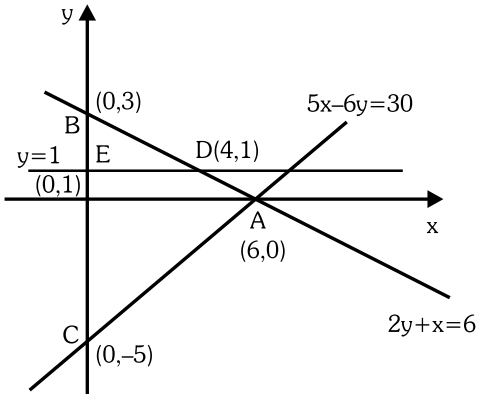
17. If a point A(x, y) lies in the region bounded by the y-axis, straight lines 2y + x = 6 and 5x – 6y = 30, then the probability that y < 1 is :

(A) $\frac{1}{6}$ (B) $\frac{5}{6}$

(C) $\frac{2}{3}$ (D) $\frac{6}{7}$

Official Ans. by NTA (B)
Allen Ans. (B)

Sol. Required probability = $\frac{ar(ADEC)}{ar(ABC)}$



$= 1 - \frac{ar(BDE)}{ar(ABC)}$

$= 1 - \frac{\frac{1}{2}\times 2\times 4}{\frac{1}{2}\times 8\times 6} = 1 - \frac{1}{6} = \frac{5}{6}$

18. The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is

(A) $\frac{26}{25}$ (B) $\frac{25}{26}$

(C) $\frac{50}{51}$ (D) $\frac{52}{51}$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $\tan^{-1}\frac{1}{1+n+n^2} = \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$= \tan^{-1}(n+1) - \tan^{-1} n$

so, $\sum_{n=1}^{50}(\tan^{-1}(n+1) - \tan^{-1} n)$

$= \tan^{-1} 51 - \tan^{-1} 1$

$\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right) = \cot(\tan^{-1} 51 + \tan^{-1} 1)$

$= \frac{1}{\tan(\tan^{-1} 51 - \tan^{-1} 1)} = \frac{1+51\times 1}{51-1} = \frac{52}{50} = \frac{26}{25}$

19. $\alpha = \sin 36°$ is a root of which of the following equation

(A) $10x^4 - 10x^2 - 5 = 0$ (B) $16x^4 + 20x^2 - 5 = 0$

(C) $16x^4 - 20x^2 + 5 = 0$ (D) $16x^4 - 10x^2 + 5 = 0$

Official Ans. by NTA (C)
Allen Ans. (C)

Sol. $\cos 72° = \frac{\sqrt{5}-1}{4}$

$\Rightarrow 1 - 2\sin^2 36° = \frac{\sqrt{5}-1}{4}$

$\Rightarrow 4 - 8\alpha^2 = \sqrt{5} - 1$

$\Rightarrow 5 - 8\alpha^2 = \sqrt{5}$

$\Rightarrow (5 - 8\alpha^2)^2 = 5$

$\Rightarrow 25 + 64\alpha^4 - 80\alpha^2 = 5$

$\Rightarrow 64\alpha^4 - 80\alpha^2 + 20 = 0$

$\Rightarrow 16\alpha^4 - 20\alpha^2 + 5 = 0$

20. Which of the following statement is a tautology?

(A) $((\sim q)\wedge p)\wedge q$

(B) $((\sim q)\wedge p)\wedge(p\wedge(\sim p))$

(C) $((\sim q)\wedge p)\vee(p\vee(\sim p))$

(D) $(p\wedge q)\wedge(\sim(p\wedge q))$

Official Ans. by NTA (C)
Allen Ans. (C)

Sol. (A) $(\sim q\wedge p)\wedge q = (\sim q\wedge q)\wedge p = f$

(B) $(\sim q\wedge p)\wedge(p\wedge\sim p) = \sim q\wedge(p\wedge\sim p) = f$

(C) $(\sim q\wedge p)\vee(p\vee\sim p) = (\sim q\wedge p)\vee(t) = t$

(D) $(p\wedge q)\wedge(\sim(p\wedge q)) = f$

SECTION-B

1. Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Define

$f : S \to S$ as $f(n) = \begin{cases} 2n, & \text{if } n = 1,2,3,4,5 \\ 2n-11 & \text{if } n = 6,7,8,9,10 \end{cases}$.

Let g : S → S be a function such that $fog(n) = \begin{cases} n+1 & \text{, if n is odd} \\ n-1 & \text{, if n is even} \end{cases}$, then

g(10) ((g(1) + g(2) + g(3) + g(4) + g(5)) is equal to:

Official Ans. by NTA (190)

Allen Ans. (190)

Sol. $f^{-1}(n)=\begin{cases}\frac{n}{2} & ; \quad n=2,4,6,8,10\\ \frac{n+11}{2} & ; \quad n=1,3,5,7,9\end{cases}$

$f(g(n))=\begin{cases}n+1 & ; \quad n\in\text{odd}\\ n-1 & ; \quad n\in\text{even}\end{cases}$

$\Rightarrow \quad g(n)=\begin{cases}f^{-1}(n+1) & ; \quad n\in\text{odd}\\ f^{-1}(n-1) & ; \quad n\in\text{even}\end{cases}$

$\therefore \quad g(n)=\begin{cases}\frac{n+1}{2} & ; \quad n\in\text{odd}\\ \frac{n+10}{2} & ; \quad n\in\text{even}\end{cases}$

$g(10)\cdot[g(1)+g(2)+g(3)+g(4)+g(5)]$

$=10\cdot[1+6+2+7+3]=190$

2. Let α, β be the roots of the equation $x^2-4\lambda x+5=0$ and α, γ be the roots of the equation $x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+7+3\lambda\sqrt{3}=0$.

If $\beta+\gamma=3\sqrt{2}$, then $(\alpha+2\beta+\gamma)^2$ is equal to :

Official Ans. by NTA (98)

Allen Ans. (98)

Sol. $x^2-4\lambda x+5=0\left\langle\begin{matrix}\alpha\\ \beta\end{matrix}\right.$

$x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+\left(7+3\lambda\sqrt{3}\right)=0\left\langle\begin{matrix}\alpha\\ \gamma\end{matrix}\right.$

$\alpha+\beta=4\lambda$

$\alpha+\gamma=3\sqrt{2}+2\sqrt{3}$

$\beta+\lambda=3\sqrt{2}$ $\qquad$ $\alpha\gamma=7+3\lambda\sqrt{3}$

$\therefore \quad \alpha=2\lambda+\sqrt{3}$ $\qquad$ $\alpha\beta=5$

$\beta=2\lambda-\sqrt{3}$ $\qquad$ $4\lambda^2=8\Rightarrow \lambda=\sqrt{2}$

$\therefore \quad (\alpha+2\beta+\lambda)^2=\left(4\alpha+3\sqrt{2}\right)^2=\left(7\sqrt{2}\right)^2=98$

3. Let A be a matrix of order 2 × 2, whose entries are from the set {0, 1, 2, 3, 4, 5}. If the sum of all the entries of A is a prime number p, 2 < p < 8, then the number of such matrices A is :

Official Ans. by NTA (180)

Allen Ans. (180)

Sol. Let $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$; a, b, c, d ∈ {0, 1, 2, 3, 4, 5}

a + b + c + d = p, p ∈ {3, 5, 7}

Case-(i)

a + b + c + d = 3; a, b, c, d ∈ {0, 1, 2, 3}

No. of ways = ${}^{3+4-1}C_{4-1}={}^{6}C_3=56$ ..... (1)

Case-(ii)

a + b + c + d = 5; a, b, c, d ∈ {0, 1, 2, 3, 4, 5}

No. of ways = ${}^{5+4-1}C_{4-1}={}^{8}C_3=56$ ..... (2)

Case-(iii)

a + b + c + d = 7

No. of ways = total ways when a, b, c, d ∈ {0, 1, 2, 3, 4, 5, 6, 7}– total ways when a, b, c, d ∉ {6, 7}

No of ways $={}^{7+4-1}C_{4-1}=\left(\frac{\lfloor 4}{\lfloor 3}+\frac{\lfloor 4}{\lfloor 2}\right)$

$={}^{10}C_3-16=104$ ..... (3)

Hence total no. of ways = 180

4. If the sum of the coefficients of all the positive powers of x, in the binomial expansion of $\left(x^n+\frac{2}{x^5}\right)^7$ is 939, then the sum of all the possible integral values of n is :

Official Ans. by NTA (57)

Allen Ans. (57)

Sol. coefficients and there cumulative sum are :

Coefficient	Commulative sum
$x^{7n}\to{}^7C_0$	1
$x^{6n-5}\to 2\cdot{}^7C_1$	$1+14$
$x^{5n-10}\to 2^2\cdot{}^7C_2$	$1+14+84$
$x^{4n-15}\to 2^3\cdot{}^7C_3$	$1+14+84+280$
$x^{3n-20}\to 2^4\cdot{}^7C_4$	$1+4+84+280+560=939$
$x^{2n-25}\to 2^5\cdot{}^7C_5$	

$3n-20\ge 0\ \cap\ 2n-25<0\ \cap\ n\in I$

$\therefore \quad 7\le n\le 12$

Sum = 7 + 8 + 9 + 10 + 11 + 12 = 57

5. Let [t] denote the greatest integer ≤ t and {t} denote the fractional part of t. Then integral value of α for which the left hand limit of the function

$$f(x)=[1+x]+\frac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$$ at x = 0 is equal to

$\alpha-\frac{4}{3}$ is ____

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $f(x)=[1+x]+\frac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$

$\lim_{x\to 0^-} f(x)=\alpha-\frac{4}{3} \Rightarrow 0+\frac{\alpha^{-1}-2}{-1}=\alpha-\frac{4}{3}$

$\Rightarrow 2-\frac{1}{\alpha}=\alpha-\frac{4}{3}$

$\Rightarrow \alpha+\frac{1}{\alpha}=\frac{10}{3}$

$\Rightarrow \alpha = 3;\ \alpha \in I$

6. If $y(x)=(x^{x^x}),\ x>0$ then $\frac{d^2x}{dy^2}+20$ at x = 1 is equal to:

Official Ans. by NTA (16)

Allen Ans. (16)

Sol. $y = (x) = (x^x)^x$

$ln\ y(x) = x^2 \cdot ln\ x$

$\frac{1}{y(x)} \cdot y'(x) = \frac{x^2}{x} + 2x \cdot ln\ x$

$y'(x) = y(x)\ [x + 2x\ ln\ x]$

$y(1) = 1;\ y'(1) = 1$

$y''(x) = y'(x)\ [x + 2x \cdot ln\ (x)]$
$\quad + y(x)\ [1 + 2(1 + ln\ x)]$

$y''(1) = 1\ [1 + 0] + 1\ (1 + 2) = 4$

$\frac{d^2y}{dx^2} = -\left(\frac{dy}{dx}\right)^3 \cdot \frac{d^2x}{dy^2}$

$\Rightarrow 4 = -\frac{d^2x}{dy^2}$

$\frac{d^2x}{dy^2} = -4$

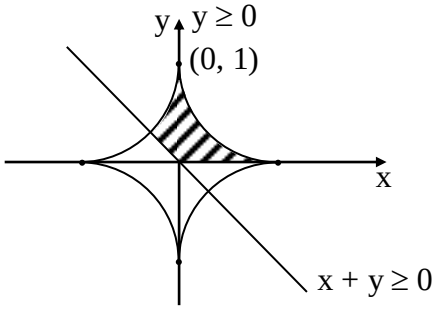
Ans. –4 + 20 = 16

7. If the area of the region $\left\{(x,y): x^{\frac{2}{3}}+y^{\frac{2}{3}}\le 1\, x+y\ge 0,\ y\ge 0\right\}$ is A, then $\frac{256A}{\pi}$ is

Official Ans. by NTA (36)

Allen Ans. (36)

Sol.



$A=\frac{3}{2}\int_0^1 (1-x^{2/3})^{3/2}dx$

Let $x = \sin^3\theta$

$A=\frac{3}{2}\int_0^{\pi/2}\left(1-\sin^2\theta\right)^{3/2}.3\sin^2\theta\cos\theta\, d\theta$

$=\frac{3}{2}\int_0^{\pi/2} 3\sin^2\theta\cos^4\theta\, d\theta$

$=\frac{9}{2}\int_0^{\pi/2} \sin^2\theta\cos^4\theta\, d\theta$

$A=\frac{9}{2}\times\frac{1.3.1}{(2+4)(4)(2)}.\frac{\pi}{2}$

$\Rightarrow A=\frac{9\pi}{64}\Rightarrow\frac{64A}{\pi}=9$

$\Rightarrow\frac{256A}{\pi}=36$ **Ans.**

8. Let v be the solution of the differential equation $(1-x^2)dy=\left(xy+(x^3+2)\sqrt{1-x^2}\right)dx,\ -1<x<1$

and y (0) = 0 if $\int_{-\frac{1}{2}}^{\frac{1}{2}}\sqrt{1-x^2}\,y(x)dx=k$ then k^{-1} is equal to :

Official Ans. by NTA (320)

Allen Ans. (320)

Sol. $(1-x^2)\frac{dy}{dx} = xy + (x^3+2)\sqrt{1-x^2}$

$\Rightarrow \frac{dy}{dx}+\left(\frac{-x}{1-x^2}\right)y=\frac{x^3+2}{\sqrt{1-x^2}}$

$IF = e^{\int\frac{-x}{1-x^2}dx} = \sqrt{1-x^2}$

$y(x)\cdot\sqrt{1-x^2} = \frac{x^4}{4} + 2x + c$

$y(0) = 0 \Rightarrow c = 0$

$\sqrt{1-x^2}\,y(x) = \frac{x^4}{4} + 2x$

required value $= \int_{-1/2}^{1/2}\left(\frac{x^4}{4}+2x\right)dx - \frac{1}{4}\cdot 2\int_0^{1/2} x^4 dx$

$= \frac{1}{10}\left(x^5\right)_0^{1/2} = \frac{1}{320}$

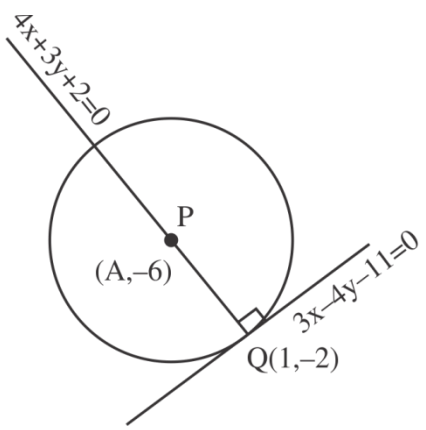
$k^{-1} = 320$

9. Let a circle C of radius 5 lie below the x-axis. The line $L_1 = 4x+3y-2$ passes through the centre P of the circle C and intersects the line $L_2 : 3x-4y-11=0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x-12y+51=0$ is

Official Ans. by NTA (11)

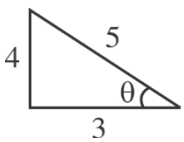
Allen Ans. (11)

Sol.



$4x + 3y + 2 = 0$

$3x - 4y - 11 = 0$



$\frac{x}{-25} = \frac{y}{50} = \frac{1}{-25}$

$\frac{x-1}{\cos\theta} = \frac{y+2}{\sin\theta} = \pm 5$

$y = -2 + 5\left(-\frac{4}{5}\right) = -6$

$x = 1 + 5\left(\frac{3}{5}\right) = 4$

Req. distance

$\left|\frac{5(4)-12(-6)+51}{13}\right|$

$= \left|\frac{20+72+51}{13}\right|$

$= \frac{143}{13} = 11$

10. Let $S = \{E, E_2 .... E_8\}$ be a sample space of random experiment such that $P(E_n) = \frac{n}{36}$ for every n = 1, 2….8. Then the number of elements in the set $\left\{A \subset S : P(A) \ge \frac{4}{5}\right\}$ is _____

Official Ans. by NTA (19)

Allen Ans. (19)

Sol. $P(A') < \frac{1}{5} = \frac{36}{180}$

5 times the sum of missing number should be less than 36.

If 1 digit is missing = 7

If 2 digit is missing = 9

If 3 digit is missing = 2

If 0 digit is missing = 1

Alternate

A is subset of S hence

A can have elements:

type 1 : { }

type 2: $\{E_1\}$, $\{E_2\}$, ... $\{E_8\}$

type 3: $\{E_1, E_2\}$, $\{E_1, E_3\}$..... $\{E_1, E_8\}$

⋮

type 6: $\{E_1, E_2, ... E_5\}$, ...$\{E_4, E_5, E_6, E_7, E_8\}$

type 7: $\{E_1, E_2, ... E_6\}$, ...$\{E_3, E_4, ... E_8\}$

type 8: $\{E_1, E_2, ... E_7\}$ $\{E_2, E_3, ... E_8\}$

type 9: $\{E_1, E_2, ... E_8\}$

As $P(A) \geq \frac{4}{5}$;

Note : Type 1 to Type 4 elements can not be in set A as maximum probability of type 4 elements.

$\{E_5, E_6, E_7, E_8\}$ is $\frac{5}{36}+\frac{6}{36}+\frac{7}{36}+\frac{8}{36}=\frac{13}{18}<\frac{4}{5}$

Now for Type 5 acceptable elements let's call probability as P_5

$$P_5 = \frac{n_1+n_2+n_3+n_4+n_5}{36} \leq \frac{4}{5}$$

$\Rightarrow n_1 + n_2 + n_3 + n_4 + n_5 \geq 28.8$

Hence, 2 possible ways $\{E_5, E_6, E_7, E_8,$ **E_3 or E_4**$\}$

$P_6 = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 \geq 28.8$

$\Rightarrow$ 9 possible ways

$P_7 \Rightarrow n_1 + n_2 + ... + n_7 \geq 288$

$\Rightarrow$ 7 possible ways

$P_8 \Rightarrow n_1 + n_2 + ... + n_8 \geq 28.8$

$\Rightarrow$ 1 possible way

Total = 19