



JEE-MAIN – JUNE, 2022

(Held On Tuesday 27th June, 2022)

TIME : 9 : 00 AM to 12 : 00 PM

Mathematics

Test Pattern : JEE-MAIN

Maximum Marks : 120

Topic Covered: FULL SYLLABUS

Important instruction:

1. Use Blue / Black Ball point pen only.
2. There are three sections of equal weightage in the question paper **Physics, Chemistry** and **Mathematics** having 30 questions in each subject. Each paper have 2 sections A and B.
3. You are awarded +4 marks for each correct answer and -1 marks for each incorrect answer.
4. Use of calculator and other electronic devices is not allowed during the exam.
5. No extra sheets will be provided for any kind of work.

Name of the Candidate (in Capitals) _____

Father's Name (in Capitals) _____

Form Number : in figures _____

: in words _____

Centre of Examination (in Capitals): _____

Candidate's Signature: _____ Invigilator's Signature : _____

Rough Space

YOUR TARGET IS TO SECURE GOOD RANK IN JEE-MAIN

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FINAL JEE-MAIN EXAMINATION – JUNE, 2022

(Held On Monday 27th June, 2022)

TIME : 9 : 00 AM to 12 : 00 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z} = iz^2$ is :

- (A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$
 (C) $\frac{3}{2}$ (D) $\frac{3}{4}$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. \Rightarrow Let $z = x + iy$, $x, y \in \mathbb{R}$

$$\text{Now } \bar{z} = iz^2$$

$$\text{then } x - iy = i(x^2 - y^2 + 2xyi)$$

$$x - iy = i(x^2 - y^2) - 2xy$$

$$\Rightarrow x = -2xy \text{ & } -y = x^2 - y^2$$

$$\Rightarrow x(1 + 2y) = 0$$

$$x = 0 \text{ or } y = -\frac{1}{2}$$

$$\text{Put } x = 0 \text{ in } -y = x^2 - y^2$$

$$\text{We get } y = y^2$$

$$\Rightarrow y = 0, 1$$

Similarly

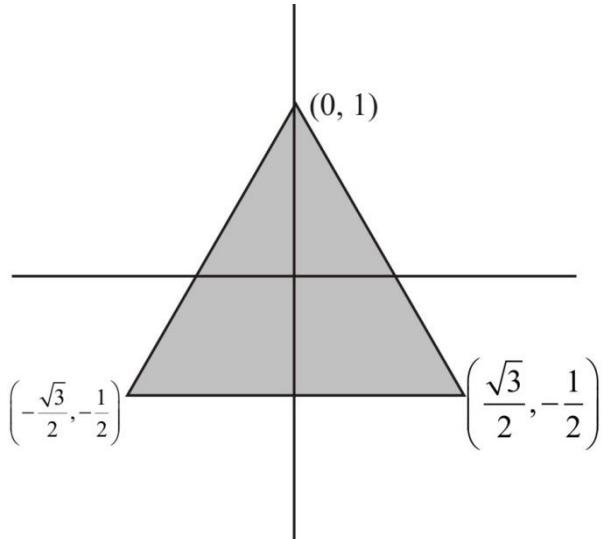
$$\text{Put } y = -\frac{1}{2} \text{ in } -y = x^2 - y^2$$

$$\Rightarrow \frac{1}{2} = x^2 - \frac{1}{4}$$

$$\Rightarrow x^2 = \frac{3}{4}$$

$$x = \pm \frac{\sqrt{3}}{2}$$

$$z = \left(0, i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i \right)$$



$$\begin{aligned} \text{Area} &= \frac{1}{2} \cdot \left(\sqrt{3}\right) \left(\frac{3}{2}\right) \\ &= \frac{3\sqrt{3}}{4} \end{aligned}$$

2. Let the system of linear equations $x + 2y + z = 2$, $\alpha x + 3y - z = \alpha$, $-\alpha x + y + 2z = -\alpha$ be inconsistent. Then α is equal to :

- (A) $\frac{5}{2}$ (B) $-\frac{5}{2}$
 (C) $\frac{7}{2}$ (D) $-\frac{7}{2}$

Official Ans. by NTA (D)

Allen Ans. (D)

$$\text{Sol. } \Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ -2 & 1 & 2 \end{vmatrix}$$

$$\begin{aligned} &= (6 + y) - 2((2\alpha - \alpha) + 1(\alpha + 3\alpha)) \\ &= 7 - 2\alpha + 4\alpha \\ &= 7 + 2\alpha \end{aligned}$$

$$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix}$$

$$= 14 + 2\alpha$$

$$\alpha = -x_2 = 7$$

$$\Delta_1 \neq 0$$

3. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$, where a, b, c

are in A.P. and $|a| < 1, |b| < 1, |c| < 1, abc \neq 0$, then

(A) x, y, z are in A.P.

(B) x, y, z are in G.P.

(C) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

(D) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a+b+c)$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $x = 1 + a + a^2 = \dots\dots\dots$

$$x = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$$

$$y = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$$

$$z = \frac{1}{1-c} \Rightarrow c = 1 - \frac{1}{z}$$

a, b, c are in A.P.

$\Rightarrow 1 - \frac{1}{x}, 1 - \frac{1}{y}, 1 - \frac{1}{z}$ are in A.P.

$\Rightarrow -\frac{1}{x}, -\frac{1}{y}, -\frac{1}{z}$ are in A.P.

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

4. Let $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$, where a, b, c are constants,

represent a circle passing through the point $(2, 5)$.

Then the shortest distance of the point $(11, 6)$ from this circle is :

(A) 10 (B) 8

(C) 7 (D) 5

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. Let equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x+2g)}{(2y+2f)}$$

$$\text{Comparing with } \frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$$

$$\Rightarrow b = 0, a = -2, c = 2$$

$$\Rightarrow -2g = -2 \Rightarrow g = 1 \quad 2f = -2$$

$$f = -1$$

Now circle will be

$$x^2 + y^2 + 2x - 2y + c = 0$$

its passes through $(2, 5)$

which will give $c = -23$

so circle will be $x^2 + y^2 + 2x - 2y - 23 = 0$

centre $C = (-1, 1)$

and radius 5

Now P is $(11, 6)$

So minimum distance of P from circle will be

$$= \sqrt{(11+1)^2 + (6-1)^2} - 5$$

$$= 13 - 5$$

$$= 8$$

5. Let a be an integer such that $\lim_{x \rightarrow 7} \frac{18 - [1-x]}{[x-3a]}$

exists, where $[t]$ is greatest integer $\leq t$. Then a is equal to :

(A) -6 (B) -2

(C) 2 (D) 6

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $\lim_{x \rightarrow 7^-} \frac{18 - [1-x]}{[x] - 3a}$

L.H.L. $\lim_{x \rightarrow 7^-} \frac{18 - [1-x]}{[x] - 3a}$

$$= \frac{18 - (-6)}{6 - 3a}$$

$$= \frac{24}{6 - 3a}$$

R.H.L. $\lim_{x \rightarrow 7^+} \frac{18 - [1-x]}{[x] - 3a}$

$$= \frac{18 - (-7)}{7 - 3a}$$

$$= \frac{25}{7 - 3a}$$

Now L.H.L. = R.H.L.

$$\frac{24}{6 - 3a} = \frac{25}{7 - 3a}$$

$$\Rightarrow 168 - 72a = 150 - 75a$$

$$\Rightarrow 18 = -3a$$

$$\Rightarrow a = -6$$

6. The number of distinct real roots of $x^4 - 4x + 1 = 0$ is :

- (A) 4 (B) 2
 (C) 1 (D) 0

Official Ans. by NTA (B)**Allen Ans. (B)**

Sol. Let $f(x) = x^4 - 4x + 1$

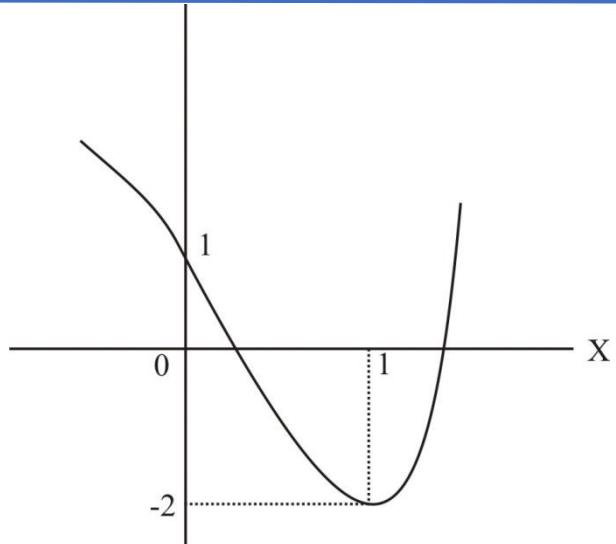
$$f(x) = 4x^3 - 4$$

$$f'(x) = 0 \Rightarrow x = 1$$

$x = 1$ is point of minima.

$$f(1) = -2$$

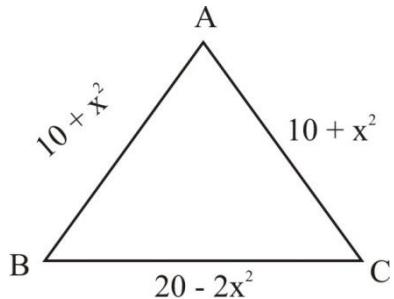
$$f(0) = 1$$



Hence 2 solutions.

7. The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 2x^2$. If for $x = k$, the area of the triangle is maximum, then $3k^2$ is equal to :

- (A) 5 (B) 8
 (C) 10 (D) 12

Official Ans. by NTA (C)**Allen Ans. (C)****Sol.**

$$a = 20 - 2x^2, b = 10 + x^2, c = 10 + x^2$$

$$= \frac{a+b+c}{2}$$

$$= 20$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{20(2x^2)(10-x^2)(10-x^2)}$$

$$= 2\sqrt{10} \sqrt{x^2(10-x^2)^2}$$

$$= 2\sqrt{10} |x(10-x^2)|$$

$$= 2\sqrt{10} |10x - x^3|$$

$$S = 10x - x^3$$

$$\frac{ds}{dx} = 10 - 3x^2$$

$$\frac{ds}{dx} = 0 \Rightarrow x^2 = \frac{10}{3}$$

$$3x^2 = 10$$

8. If $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$, $|y| < 2$, then :

(A) $x^2 y'' + xy' - 25y = 0$

(B) $x^2 y'' - xy' - 25y = 0$

(C) $x^2 y'' - xy' + 25y = 0$

(D) $x^2 y'' + xy' + 25y = 0$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$

$$\cos^{-1}\left(\frac{y}{2}\right) = 5 \log_e\left(\frac{x}{5}\right)$$

$$\frac{-1}{\sqrt{1 - \frac{y^2}{4}}} \cdot \frac{y'}{2} = 5 \cdot \frac{1}{x} \times \frac{1}{5}$$

$$\Rightarrow \frac{-y'}{\sqrt{4-y^2}} = \frac{5}{x}$$

$$-xy' = 5\sqrt{4-y^2}$$

$$-xy'' - y' = 5 \cdot \frac{1}{2\sqrt{4-y^2}} (-2y y')$$

$$\Rightarrow xy'' + y' = \frac{5y' \cdot y}{\sqrt{4-y^2}}$$

$$xy'' + y' = 5 \cdot \left(\frac{-5}{x}\right) y$$

$$x^2 y'' + xy' = -25y$$

9. $\int \frac{(x^2+1)e^x}{(x+1)^2} dx = f(x)e^x + C$, Where C is a constant, then $\frac{d^3 f}{dx^3}$ at x = 1 is equal to :

(A) $-\frac{3}{4}$ (B) $\frac{3}{4}$

(C) $-\frac{3}{2}$ (D) $\frac{3}{2}$

Official Ans. by NTA (B)

Allen Ans. (B)

$$\begin{aligned} \text{Sol. } & \int \left(\frac{x^2+1}{(x+1)^2} \right) e^x dx \\ &= \int \left(\frac{x^2-1+2}{(x+1)^2} \right) e^x dx \\ &= \int \left(\frac{x-1}{x+1} + \frac{2}{(x+1)^2} \right) e^x dx \\ &= \int (f(x) + f'(x)) e^x dx \\ &= f(x) e^x + c \end{aligned}$$

$$\text{Where } f(x) = \frac{x-1}{x+1}$$

$$f'(x) = \frac{2}{(x+1)^2}$$

$$f''(x) = \frac{-4}{(x+1)^3}$$

$$= \frac{12}{(x+1)^4}$$

$$f''(1) = \frac{12}{16}$$

$$= \frac{3}{4}$$

- 10.** The value of the integral $\int_{-2}^2 \frac{|x^3 + x|}{(e^{x|x|} + 1)} dx$ is equal

to :

- (A) $5e^2$ (B) $3e^{-2}$
 (C) 4 (D) 6

Official Ans. by NTA (D)

Allen Ans. (D)

$$\text{Sol. } f(x) = \frac{|x^3 + x|}{(e^{x|x|} + 1)} dx$$

$$\int_{-2}^2 f(x) dx = \int_0^2 (f(x) + f(-x)) dx$$

$$= \int_0^2 \left(\frac{|x^3 + x|}{(e^{x|x|} + 1)} + \frac{|-x^3 - x|}{(e^{-x|-x|} + 1)} \right) dx$$

$$= \int_0^2 \left(\frac{|x^3 + x|}{(e^{x|x|} + 1)} + \frac{|x^3 + x|}{(e^{-x|x|} + 1)} \right) dx$$

$$= \int_0^2 \left(\frac{x^3 + x}{(e^{x^2} + 1)} + \frac{x^3 + x}{(e^{-x^2} + 1)} \right) dx$$

$$I = \int_0^2 \left(\frac{x^3 + x}{1 + e^{x^2}} + \frac{e^{x^2}(x^3 + x)}{1 + e^{-x^2}} \right) dx$$

$$= \int_0^2 (x^3 + x) dx$$

$$= \left[\frac{x^4}{4} + \frac{x^2}{2} \right]_0^2$$

$$= 4 + 2 = 6$$

- 11.** If $\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0, x, y > 0, y(1) = 1$, then

$y(2)$ is equal to :

- (A) $2 + \log_2 3$ (B) $2 + \log_2 2$
 (C) $2 - \log_2 3$ (D) $2 - \log_2 3$

Official Ans. by NTA (D)

Allen Ans. (D)

$$\text{Sol. } \frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0,$$

$$x, y > 0, y(1) = 1, y(2) = ?$$

$$\frac{dy}{dx} = -\frac{2^x(2^y - 1)}{2^y(2^x - 1)}$$

$$\int \frac{2^y}{2^y - 1} dy = - \int \frac{2^x}{2^x - 1} dx$$

$$\frac{1}{\ln 2} \int \frac{2^y \ln 2}{2^y - 1} dy = -\frac{1}{\ln 2} \int \frac{2^x \ln 2}{2^x - 1} dx$$

$$\frac{1}{\ln 2} \ln |2^y - 1| = \frac{-1}{\ln 2} \ln |2^x - 1| + C$$

$$\text{At } x = 1, y = 1$$

Putting this values in above relation we get $C = 0$

$$\ln |2^y - 1| + \ln |2^x - 1| = 0$$

$$(2^x - 1)(2^y - 1) = 1$$

$$2^y - 1 = \frac{1}{2^x + 1}$$

$$\text{At } x = 2$$

$$2^y = \frac{1}{3} + 1 = \frac{4}{3}$$

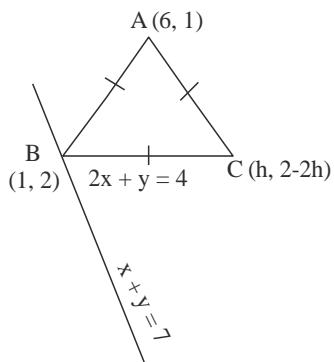
$$y = \log_2 \frac{4}{3} = \log_2 4 - \log_2 3 = 2 - \log_2 3$$

- 12.** In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is $2x + y = 4$. Let the point B lie on the line $x + 3y = 7$. If (α, β) is the centroid ΔABC , then $15(\alpha + \beta)$ is equal to :

- (A) 39 (B) 41
 (C) 51 (D) 63

Official Ans. by NTA (C)

Allen Ans. (C)

Sol.

Point B (1, 2)

Now let C be (h, 4 - 2h)

(As C lies on $2x + y = 4$) $\because \Delta$ is isosceles with base BC

$$\therefore AB = AC$$

$$\sqrt{25+1} = \sqrt{(6-h)^2 + (2h-3)^2}$$

$$\sqrt{26} = \sqrt{36+h^2-12h+4h^2+9-12h}$$

$$26 = 5h^2 - 24h + 45 \Rightarrow 5h^2 - 24h + 19 = 0$$

$$\Rightarrow 5h^2 - 5h - 19h + 19 = 0$$

$$h = \frac{19}{5} \text{ or } h = 1$$

$$\text{Thus } C\left(\frac{19}{5}, \frac{-18}{5}\right)$$

$$\text{Centroid} \left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3} \right)$$

$$\left(\frac{35+19}{15}, \frac{15-18}{15} \right)$$

$$\left(\frac{54}{15}, \frac{-3}{15} \right)$$

$$\alpha = \frac{54}{15}; \beta = \frac{-3}{15}$$

$$15(\alpha + \beta) = 51$$

13. Let the eccentricity of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b, \text{ be } \frac{1}{4}. \text{ If this ellipse passes}$$

through the point $\left(-4\sqrt{\frac{2}{5}}, 3\right)$, then $a^2 + b^2$ is equal

to :

$$(A) 29 \quad (B) 31$$

$$(C) 32 \quad (D) 34$$

Official Ans. by NTA (B)**Allen Ans. (B)**

$$\text{Sol. } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a > b$$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{1}{16} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = 1 - \frac{1}{16} = \frac{15}{16} \Rightarrow b^2 = \frac{15}{16}a^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{16 \times \frac{2}{5}}{a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{\frac{15}{16}a^2} = 1$$

$$\frac{80}{5a^2} = 1$$

$$16 = a^2$$

$$b^2 = 15$$

14. If two straight lines whose direction cosines are given by the relations $l + m - n = 0$, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is :
 (A) 6 (B) 4
 (C) 3 (D) 2

Official Ans. by NTA (A)**Allen Ans. (A)**

Sol. $l + m - n = 0$

$$3l^2 + m^2 + cl(l + m) = 0$$

$$n = l + m$$

$$3l^2 + m^2 + cl^2 + clm = 0$$

$$(3 + c)l^2 + clm + m^2 = 0$$

$$(3+c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0 \dots\dots(1)$$

\therefore lines are parallel.

Roots of (1) must be equal

$$\Rightarrow D = 0$$

$$c^2 - 4(3 + c) = 0$$

$$c^2 - 4c - 12 = 0$$

$$(c - 6)(c + 2) = 0$$

$$c = 6 \text{ or } c = -2$$

$$+\text{ve value of } c = 6$$

15. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors \vec{b} such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, \dots, 10\}$ is :

- (A) 0 (B) 1
 (C) 2 (D) 3

Official Ans. by NTA (A)**Allen Ans. (A)**

Sol. $\vec{a} = i + j - k$

$$\vec{c} = 2i - 3j + 2k$$

$$\vec{b} \times \vec{c} = \vec{a}$$

$$|\vec{b}| \in \{1, 2, \dots, 10\}$$

$$\therefore \vec{b} \times \vec{c} = \vec{a}$$

$\Rightarrow \vec{a}$ is perpendicular to \vec{b} as well as \vec{a} is perpendicular to \vec{c}

$$\text{Now } \vec{a} \cdot \vec{c} = 2 - 3 - 2 = -3 \neq 0$$

This $\vec{b} \times \vec{c} = \vec{a}$ is not possible.

$$\text{No. of vectors } \vec{b} = 0$$

16. Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3, ..., 18 and are arranged in the increasing order ($x_1 < x_2 < x_3 < x_4 < x_5$). The probability that $x_2 = 7$ and $x_4 = 11$ is :

(A) $\frac{1}{136}$ (B) $\frac{1}{72}$

(C) $\frac{1}{68}$ (D) $\frac{1}{34}$

Official Ans. by NTA (C)**Allen Ans. (C)**

- Sol. No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 from 1, 2, 3, ..., 18

$$n(s) = {}^{18}C_5$$

$$x_1 \quad (x_2) \quad x_3 \quad (x_4) \quad x_5 \\ 7 \qquad \qquad \qquad 11$$

$$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$$

$$P(E) = \frac{6 \times 3 \times 7}{{}^{18}C_5}$$

$$\frac{1}{17 \times 4} = \frac{1}{68}$$

17. Let X be a random variable having binomial distribution $B(7, p)$. If $P(X = 3) = 5P(X = 4)$, then the sum of the mean and the variance of X is :

(A) $\frac{105}{16}$ (B) $\frac{7}{16}$

(C) $\frac{77}{36}$ (D) $\frac{49}{16}$

Official Ans. by NTA (C)**Allen Ans. (C)**

Sol. B (7, p)

$$n = 7 \quad p = p$$

given

$$P(x=3) = 5P(x=4)$$

$${}^7C_3 \times p^3 (1-p)^4 = 5 \cdot {}^7C_4 p^4 (1-p)^3$$

$$\frac{{}^7C_3}{5 \times {}^7C_4} = \frac{p}{1-p}$$

$$1-p = 5p$$

$$6p = 1$$

$$p = \frac{1}{6} \Rightarrow q = \frac{5}{6}$$

$$n = 7$$

$$\text{Mean} = np = 7 \times \frac{1}{6} = \frac{7}{6}$$

$$\text{Var} = npq = 7 \times \frac{1}{6} \times \frac{5}{6} = \frac{35}{36}$$

Sum

$$= \frac{7}{6} + \frac{35}{36}$$

$$= \frac{42+35}{36}$$

$$= \frac{77}{36}$$

18. The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$

is equal to :

- | | |
|--------------------|--------------------|
| (A) -1 | (B) $-\frac{1}{2}$ |
| (C) $-\frac{1}{3}$ | (D) $-\frac{1}{4}$ |

Official Ans. by NTA (B)

Allen Ans. (B)

$$\text{Sol. } \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$$

$$= \frac{\sin\left(3 \times \frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \times \cos\left(\frac{\frac{2\pi}{7} + \frac{6\pi}{7}}{2}\right)$$

$$= \frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right)$$

$$= \frac{\sin\left(\frac{7\pi}{7}\right) + \sin\left(\frac{-\pi}{7}\right)}{2\sin\frac{\pi}{7}}$$

$$= \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}$$

$$= -\frac{1}{2}$$

19. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to :

$$(A) \frac{11\pi}{12} \quad (B) \frac{17\pi}{12}$$

$$(C) \frac{31\pi}{12} \quad (D) -\frac{3\pi}{4}$$

Official Ans. by NTA (A)

Allen Ans. (A)

$$\text{Sol. } \sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\tan\left(\frac{3\pi}{4}\right)$$

$$\sin^{-1}\sin\left(\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\cos^{-1}\left(\cos\frac{2\pi}{6}\right) = 2\pi - \frac{7\pi}{6} = \frac{5\pi}{6}$$

$$\tan^{-1}\tan\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} - \pi = -\frac{\pi}{4}$$

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\cos\frac{7\pi}{6} + \tan^{-1}\tan\frac{3\pi}{4}$$

$$= \frac{11\pi}{12}$$

20. The Boolean expression $(\sim(p \wedge q)) \vee q$ is equivalent to :

- (A) $q \rightarrow (p \wedge q)$ (B) $p \rightarrow q$
 (C) $p \rightarrow (p \rightarrow q)$ (D) $p \rightarrow (p \vee q)$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $\begin{aligned} & (\sim(p \wedge q)) \vee q \\ & = (\sim p \vee \sim q) \vee q \\ & = \sim p \vee \sim q \vee q \\ & = \sim p \vee t \\ & = \text{this statement is a tautology option D} \\ & p \Rightarrow (p \vee q) \text{ is also a tautology.} \end{aligned}$

OR

p	q	$P \wedge q$	$\sim(P \wedge q)$	$\sim(P \wedge q) \vee q$	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	F	T

SECTION-B

1. Let $f : R \rightarrow R$ be a function defined $f(x) = \frac{2e^{2x}}{e^{2x} + e}$.

Then $f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \dots + f\left(\frac{99}{100}\right)$ is equal to _____.

Official Ans. by NTA (99)

Allen Ans. (99)

Sol.

$$\begin{aligned} f(x) + f(1-x) &= \frac{2e^{2x}}{e^{2x} + e} + \frac{2e^{2-2x}}{e^{2-2x} + e} = \left[\frac{e^{2x}}{e^{2x} + e} + \frac{e^2}{e^2 + e^{2x+1}} \right] \\ &= 2 \left[\frac{e^{2x-1}}{e^{2x-1} + 1} + \frac{1}{1 + e^{2x-1}} \right] = 2 \end{aligned}$$

$$f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \dots + f\left(\frac{99}{100}\right)$$

$$= \left\{ f\left(\frac{1}{100}\right) + f\left(\frac{99}{100}\right) \right\} + \left\{ f\left(\frac{2}{100}\right) + f\left(\frac{98}{100}\right) \right\} + \dots + f\left(\left[\frac{49}{100}\right] + f\left(\frac{51}{100}\right)\right) + f\left(\frac{1}{2}\right)$$

$$\begin{aligned} &= (2 + 2 + 2 + \dots - 49 \text{ times}) + \frac{2e}{e + e} \\ &= 98 + 1 = 99 \end{aligned}$$

2. If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e P$, then P is equal to _____.

Official Ans. by NTA (45)

Allen Ans. (45)

Sol. $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$

$$(e^x)^3 - 11(e^x)^2 - 45 + \frac{81e^x}{2} = 0$$

$$e^x = t$$

$$2t^3 - 22t^2 + 81t - 90 = 0$$

$$t_1 t_2 t_3 = 45$$

$$e^{x_1} \cdot e^{x_2} \cdot e^{x_3} = 45$$

$$e^{x_1+x_2+x_3} = 45$$

$$\log_e e^{x_1+x_2+x_3} = \log_e 45$$

$$x_1 + x_2 + x_3 = \log_e 45$$

$$\log_e P = \log_e 45$$

$$P = 45$$

3. The positive value of the determinant of the matrix

$$\text{A, whose } \text{Adj}(\text{Adj}(A)) = \begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix},$$

is _____.

Official Ans. by NTA (14)

Allen Ans. (14)

Sol. $Adj(AdjA) = \begin{bmatrix} 14 & 18 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix}$

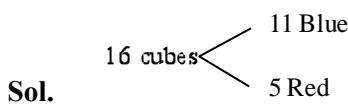
$$|Adj(AdjA)| = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix} = 14 \times 14 \times 14 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = (14)^3 [3 - 2(-5) - 1(-1)] = (14)^3 [14] = (14)^4$$

$$|A|^4 = (14)^4 \Rightarrow |A| = 14$$

4. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is _____.

Official Ans. by NTA (56)

Allen Ans. (56)



$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$$

$$x_1, x_6 \geq 0, \quad x_2, x_3, x_4, x_5 \geq 2$$

$$x_2 = t_1 + 2$$

$$x_3 = t_2 + 2$$

$$x_4 = t_3 + 2$$

$$x_5 = t_4 + 2$$

$$x_1, t_2, t_3, t_4, t_5, x_6 \geq 0$$

$$\text{No. of solutions} = {}^{6+3-1}C_3 = {}^8C_3 = 56$$

5. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^4} + \frac{\sqrt{5}}{x^3}\right)^{60}$ is $5^k l$, where $l, k \in \mathbb{N}$ and l is co-prime to 5, then k is equal to _____.

Official Ans. by NTA (5)

Allen Ans. (5)

Sol. $\left(\frac{\sqrt{x}}{5^{1/4}} + \frac{\sqrt{5}}{x^{1/3}}\right)^{60}$

$$T_{r+1} = {}^{60}C_r \left(\frac{x^{1/2}}{5^{1/4}}\right)^{60-r} \left(\frac{5^{1/2}}{x^{1/3}}\right)^r$$

$$= {}^{60}C_r 5^{\frac{3r-60}{4}} \cdot x^{\frac{180-5r}{6}}$$

$$\frac{180-5r}{6} = 10 \Rightarrow r = 24$$

$$\text{Coeff. of } x^{10} = {}^{60}C_{24} 5^3 = \frac{|60|}{|24|36} 5^3$$

$$\text{Powers of 5 in } {}^{60}C_{24} \cdot 5^3 = \frac{5^{14}}{5^4 \times 5^8} \times 5^3 = 5^5$$

6. Let

$$A_1 = \{(x, y) : |x| \leq y^2, |x| + 2y \leq 8\} \text{ and}$$

$$A_2 = \{(x, y) : |x| + |y| \leq k\}. \text{ If } 27 \text{ (Area } A_1) = 5$$

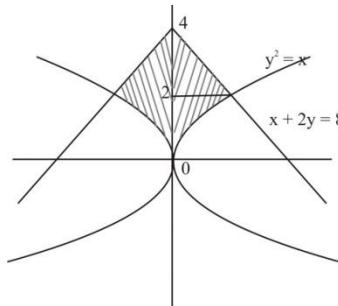
(Area A_2), then k is equal to :

Official Ans. by NTA (6)

Allen Ans. (6)

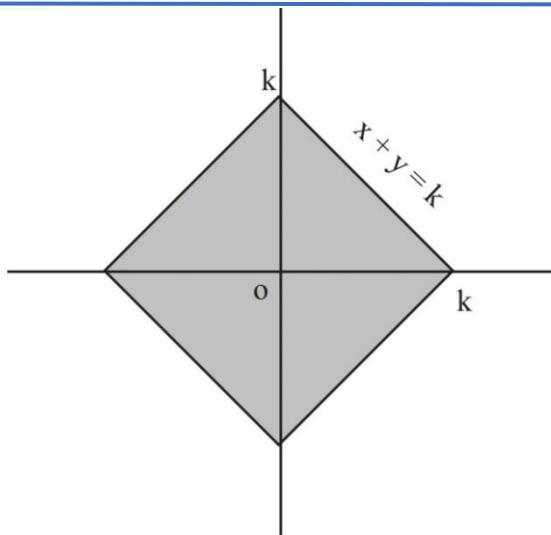
Sol. $A_1 = \{(x, y) : |x| \leq y^2, |x| + 2y \leq 8\} \text{ and}$

$$A_2 = \{(x, y) : |x| + |y| \leq k\}.$$



$$\text{area}(A_1) = 2 \left[\int_0^2 y^2 dy + \int_2^4 (8 - 2y) dy \right]$$

$$= 2 \left[\left(\frac{y^3}{3} \right)_0^2 + (8y - y^2)_2^4 \right]$$



$$\text{area}(A_1) = 2 \times \frac{20}{3} = \frac{40}{3}$$

$$\text{Area}(A_2) = 4 \times \frac{1}{2}k^2$$

$$\text{Area}(A_2) = 2k^2$$

Now

$$27 (\text{Area } A_1) = 5 (\text{Area } A_2)$$

$$9 \times 4 = k^2$$

$$k = 6$$

7. If the sum of the first ten terms of the series

$$\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \dots \text{ is } \frac{m}{n}, \text{ where}$$

m and n are co-prime numbers, then $m + n$ is equal to _____.

Official Ans. by NTA (276)

Allen Ans. (276)

$$\text{Sol. } \frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \dots$$

$$T_n = \frac{n}{4n^4 + 1}$$

$$\begin{aligned} &= \frac{n}{(2n^2 + 1)^2 - (2n)^2} = \frac{n}{(2n^2 + 2n + 1)(2n^2 - 2n + 1)} \\ &= \frac{1}{4} \left[\frac{1}{2n^2 - 2n + 1} - \frac{1}{2n^2 + 2n + 1} \right] \end{aligned}$$

$$S_{10} = \sum_{n=1}^{10} T_n = \frac{1}{4} \left[\frac{1}{1} - \frac{1}{5} + \frac{1}{5} - \frac{1}{13} + \dots + \frac{1}{200+20+1} \right]$$

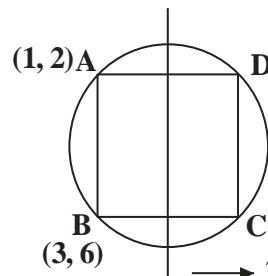
$$= \frac{1}{4} \left[1 - \frac{1}{221} \right] = \frac{1}{4} \times \frac{220}{221} - \frac{55}{221} = \frac{m}{n}$$

$$m + n = 55 + 221 = 276$$

8. A rectangle R with end points of one of its dies as $(1, 2)$ and $(3, 6)$ is inscribed in a circle. If the equation of a diameter of the circle is $2x - y + 4 = 0$, then the area of R is _____.

Official Ans. by NTA (16)

Allen Ans. (16)



Sol.

Eq. of line AB

$$y = 2x$$

Slope of AB = 2

Slope of given diameter = 2

So the diameter is parallel to AB

Distance between diameter and line AB

$$= \left(\frac{4}{\sqrt{2^2 + 1^2}} \right) = \frac{4}{\sqrt{5}}$$

$$\text{Thus } BC = 2 \times \frac{4}{\sqrt{5}} = \frac{8}{\sqrt{5}}$$

$$AB = \sqrt{(1-3)^2 + (2-6)^2} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Area} = AB \times BC = \frac{8}{\sqrt{5}} \times 2\sqrt{5} = 16 \text{ Ans.}$$

9. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$.

Then $(4\alpha - 8)^2$ is equal to _____.

Official Ans. by NTA (63)

Allen Ans. (63)

Sol. Vertex and focus of parabola $y^2 = 2x$

are $V(0, 0)$ and $S\left(\frac{1}{2}, 0\right)$ resp.

Let equation of circle be

$$(x - h)^2 + (y - k)^2 = 4$$

\therefore Circle passes through $(0, 0)$

$$\Rightarrow h^2 + k^2 = 4 \dots\dots(1)$$

\therefore Circle passes through $\left(\frac{1}{2}, 0\right)$

$$\left(\frac{1}{2} - h\right)^2 + k^2 = 4$$

$$\Rightarrow h^2 + k^2 - h = \frac{15}{4} \dots\dots(2)$$

On solving (1) and (2)

$$4 - h = \frac{15}{4}$$

$$h = 4 - \frac{15}{4} = \frac{1}{4}$$

$$k = +\frac{\sqrt{63}}{4}$$

$k = -\frac{\sqrt{63}}{4}$ is rejected as circle with centre

$\left(\frac{1}{4}, -\frac{\sqrt{63}}{4}\right)$ can't touch given parabola.

Equation of circle is

$$\left(x - \frac{1}{4}\right)^2 + \left(k - \frac{\sqrt{63}}{4}\right)^2 = 4$$

From figure

$$\alpha = 2 + \frac{\sqrt{63}}{4} = \frac{8 + \sqrt{63}}{4}$$

$$4\alpha - 8 = \sqrt{63}$$

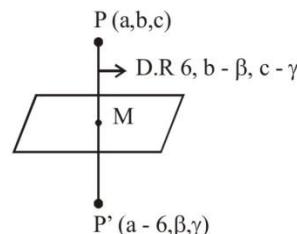
$$(4\alpha - 8)^2 = 63$$

10. Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be $(a - 6, \beta, \gamma)$. If $a + b + c = 5$, then $7\beta - 9\gamma$ is equal to _____.

Official Ans. by NTA (137)

Allen Ans. (137)

Sol.



$$M = \left(a - 3, \frac{\beta + b}{2}, \frac{\gamma + c}{2}\right)$$

Since M lies on $3x + 4y + 12z + 19 = 0$

$$\Rightarrow 6a - 4b + 12c - 4\beta + 12\gamma + 20 = 0 \dots\dots(1)$$

Since PP' is parallel to normal of the plane then

$$\frac{6}{3} = \frac{b - \beta}{-4} = \frac{c - \gamma}{12}$$

$$\Rightarrow \beta = b + 8, \quad \gamma = c - 24$$

$$a + b + c = 5 \Rightarrow a + \beta - 8 + \gamma + 24 = 5$$

$$\Rightarrow a = -\beta - \gamma - 11$$

Now putting these values in (1) we get

$$6(-\beta - \gamma - 11) - 4(\beta + b) + 12(\gamma + c) - 4\beta + 12\gamma + 20 = 0$$

$$\Rightarrow 7\beta - 9\gamma = 170 - 33 = 137$$