



JEE-MAIN – JUNE, 2022

(Held On Tuesday 28th June, 2022)

TIME : 3 : 00 PM to 6 : 00 PM

Mathematics

Test Pattern : JEE-MAIN

Maximum Marks : 120

Topic Covered: FULL SYLLABUS

Important instruction:

1. Use Blue / Black Ball point pen only.
2. There are three sections of equal weightage in the question paper **Physics, Chemistry** and **Mathematics** having 30 questions in each subject. Each paper have 2 sections A and B.
3. You are awarded +4 marks for each correct answer and -1 marks for each incorrect answer.
4. Use of calculator and other electronic devices is not allowed during the exam.
5. No extra sheets will be provided for any kind of work.

Name of the Candidate (in Capitals) _____

Father's Name (in Capitals) _____

Form Number : in figures _____

: in words _____

Centre of Examination (in Capitals): _____

Candidate's Signature: _____ Invigilator's Signature : _____

Rough Space

YOUR TARGET IS TO SECURE GOOD RANK IN JEE-MAIN

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4. The term independent of x in the expression of $(1-x^2+3x^3)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$, $x \neq 0$ is
- (A) $\frac{7}{40}$ (B) $\frac{33}{200}$
 (C) $\frac{39}{200}$ (D) $\frac{11}{50}$

Official Ans. by NTA (B)**Allen Ans. (B)**

$$\text{Sol. } (1-x^2+3x^3)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$$

General term of $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$ is

$${}^{11}C_r \left(\frac{5}{2}x^3\right)^{11-r} \left(-\frac{1}{5x^2}\right)^r$$

$$\text{General term is } {}^{11}C_r \left(\frac{5}{2}\right)^{11-r} \left(-\frac{1}{5}\right)^r x^{33-5r}$$

Now, term independent of x

$$1 \times \text{coefficient of } x^0 \text{ in } \left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$$

$$- 1 \times \text{coefficient of } x^{-2} \text{ in } \left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11} +$$

$$3 \times \text{coefficient of } x^{-3} \text{ in } \left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$$

$$\text{for coefficient of } x^0 \quad 33 - 5r = 0 \text{ not possible}$$

$$\text{for coefficient of } x^{-2} \quad 33 - 5r = -2$$

$$35 = 5r \Rightarrow r = 7$$

$$\text{for coefficient of } x^{-3} \quad 33 - 5r = -3$$

$$36 = 5r \text{ not possible}$$

So term independent of x is

$$(-1) {}^{11}C_7 \left(\frac{5}{2}\right)^4 \left(-\frac{1}{5}\right)^7 = \frac{33}{200}$$

5. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1 : 7$ and $a + n = 33$, then the value of n is
- (A) 21 (B) 22
 (C) 23 (D) 24

Official Ans. by NTA (C)**Allen Ans. (C)**

$$\text{Sol. } d = \frac{100-a}{n+1}$$

$$A_1 = a + d$$

$$A_n = 100 - d$$

$$\Rightarrow \frac{A_1}{A_n} = \frac{1}{7} \Rightarrow \frac{a+d}{100-d} = \frac{1}{7}$$

$$\Rightarrow 7a + 8d = 100$$

$$\Rightarrow 7a + 8\left(\frac{100-a}{n+1}\right) = 100 \quad \dots(1)$$

$$\because a + n = 33 \quad \dots(2)$$

Now, by Eq. (1) and (2)

$$7n^2 - 132n - 667 = 0$$

$$[n = 23] \text{ and } n = \frac{-29}{7} \text{ reject.}$$

6. Let $f, g: \mathbf{R} \rightarrow \mathbf{R}$ be functions defined by

$$f(x) = \begin{cases} [x] & , x < 0 \\ |1-x| & , x \geq 0 \end{cases} \text{ and}$$

$$g(x) = \begin{cases} e^x - x & , x < 0 \\ (x-1)^2 - 1 & , x \geq 0 \end{cases}$$

where $[x]$ denote the greatest integer less than or equal to x . Then, the function fog is discontinuous at exactly :

- (A) one point (B) two points
 (C) three points (D) four points

Official Ans. by NTA (B)**Allen Ans. (B)**

- Sol. Check continuity at $x = 0$ and also check continuity at those x where $g(x) = 0$
 $g(x) = 0$ at $x = 0, 2$

$$fog(0^+) = -1$$

$$fog(0) = 0$$

Hence, discontinuous at $x = 0$

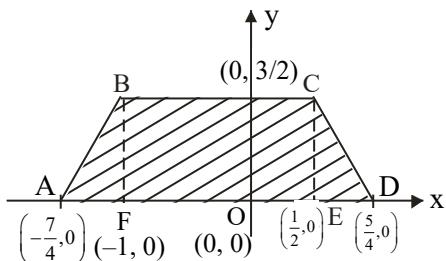
$$fog(2^+) = 1$$

$$fog(2^-) = -1$$

Hence, discontinuous at $x = 2$

Sol. $y = \begin{cases} 3 + (x+1) + \left(x - \frac{1}{2}\right), & x < -1 \\ 3 - (x+1) + \left(x - \frac{1}{2}\right), & -1 \leq x < \frac{1}{2} \\ 3 - (x+1) - \left(x - \frac{1}{2}\right), & \frac{1}{2} \leq x \end{cases}$

$$y = \begin{cases} \frac{7}{2} + 2x, & x < -1 \\ \frac{3}{2}, & -1 \leq x < \frac{1}{2} \\ \frac{5}{2} - 2x, & \frac{1}{2} \leq x \end{cases}$$



Area bounded = ar ABF + ar BCEF + ar CDE

$$\begin{aligned} &= \frac{1}{2} \left(\frac{3}{4}\right) \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right) \left(\frac{3}{2}\right) + \frac{1}{2} \left(\frac{3}{4}\right) \left(\frac{3}{2}\right) \\ &= \frac{27}{8} \text{ sq. units.} \end{aligned}$$

- 10.** Let $x = x(y)$ be the solution of the differential equation $2ye^{x/y^2}dx + (y^2 - 4xe^{x/y^2})dy = 0$ such that $x(1) = 0$. Then, $x(e)$ is equal to
 (A) $e \log_e(2)$ (B) $-e \log_e(2)$
 (C) $e^2 \log_e(2)$ (D) $-e^2 \log_e(2)$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $2ye^{x/y^2}dx + (y^2 - 4xe^{x/y^2})dy = 0$
 $2e^{x/y^2} [ydx - 2xdy] + y^2 dy = 0$

$$2e^{x/y^2} \left[\frac{y^2 dx - x \cdot (2y) dy}{y} \right] + y^2 dy = 0$$

Divide by y^3

$$2e^{x/y^2} \left[\frac{y^2 dx - x \cdot (2y) dy}{y^4} \right] + \frac{1}{y} dy = 0$$

$$2e^{x/y^2} d\left(\frac{x}{y^2}\right) + \frac{1}{y} dy = 0$$

Integrating

$$\int 2e^{x/y^2} d\left(\frac{x}{y^2}\right) + \int \frac{1}{y} dy = 0$$

$$2e^{x/y^2} + \ell ny + c = 0$$

(0, 1) lies on it.

$$2e^0 + \ell n 1 + c = 0 \Rightarrow c = -2$$

Required curve : $2e^{x/y^2} + \ell ny - 2 = 0$

For x (e)

$$2e^{x/e^2} + \ell ne - 2 = 0 \Rightarrow x = -e^2 \log_e 2$$

- 11.** Let the slope of the tangent to a curve $y = f(x)$ at (x, y) be given by $2 \tan x (\cos x - y)$. if the curve passes through the point $\left(\frac{\pi}{4}, 0\right)$, then the value

of $\int_0^{\pi/2} y dx$ is equal to

(A) $(2 - \sqrt{2}) + \frac{\pi}{\sqrt{2}}$ (B) $2 - \frac{\pi}{\sqrt{2}}$

(C) $(2 + \sqrt{2}) + \frac{\pi}{\sqrt{2}}$ (D) $2 + \frac{\pi}{\sqrt{2}}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $\frac{dy}{dx} = 2 \tan x \cos x - 2 \tan x \cdot y$

$$\frac{dy}{dx} + (2 \tan x) y = 2 \sin x$$

$$\text{Integrating factor} = e^{\int 2 \tan x dx} = \frac{1}{\cos^2 x}$$

$$y \left(\frac{1}{\cos^2 x} \right) = \int \frac{2 \sin x}{\cos^2 x} dx$$

$$y \sec^2 x = \frac{2}{\cos x} + C$$

$$y = 2 \cos x + C \cos^2 x$$

Passes through $\left(\frac{\pi}{4}, 0\right)$

$$0 = \sqrt{2} + \frac{C}{2} \Rightarrow C = -2\sqrt{2}$$

$f(x) = 2 \cos x - 2\sqrt{2} \cos^2 x$: Required curve

$$\int_0^{\pi/2} y dx = 2 \int_0^{\pi/2} \cos x dx - 2\sqrt{2} \int_0^{\pi/2} \cos^2 x dx$$

$$= [2 \sin x]_0^{\pi/2} - 2\sqrt{2} \left[\frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{\pi/2}$$

$$= 2 - \frac{\pi}{\sqrt{2}}$$

12. Let a triangle be bounded by the lines $L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ and the line L_3 , which passes through the point $P(2, 3)$, intersect L_2 at A and L_1 at B . If the point P divides the line-segment AB , internally in the ratio $1 : 3$, then the area of the triangle is equal to

(A) $\frac{110}{13}$ (B) $\frac{132}{13}$

(C) $\frac{142}{13}$ (D) $\frac{151}{13}$

Official Ans. by NTA (B)

Allen Ans. (B)

- Sol. Points A lies on L_2

$$A\left(\alpha, 4 + \frac{4}{3}\alpha\right)$$

Points B lies on L_1

$$B\left(\beta, 2 - \frac{2}{5}\beta\right)$$

Points P divides AB internally in the ratio $1 : 3$

$$\Rightarrow P(2, 3) = P\left(\frac{3\alpha + \beta}{4}, \frac{3\left(4 + \frac{4}{3}\alpha\right) + 1\left(2 - \frac{2}{5}\beta\right)}{4}\right)$$

$$\Rightarrow \alpha = \frac{3}{13}, \beta = \frac{95}{13}$$

$$\text{Point } A\left(\frac{3}{13}, \frac{56}{13}\right), B\left(\frac{95}{13}, -\frac{12}{13}\right)$$

Vertex C of triangle is the point of intersection of L_1 & L_2

$$\Rightarrow C\left(-\frac{15}{13}, \frac{32}{13}\right)$$

$$\text{area } \Delta ABC = \frac{1}{2} \begin{vmatrix} \frac{3}{13} & \frac{56}{13} & 1 \\ \frac{95}{13} & -\frac{12}{13} & 1 \\ -\frac{15}{13} & \frac{32}{13} & 1 \end{vmatrix}$$

$$= \frac{1}{2 \times 13^3} \begin{vmatrix} 3 & 56 & 13 \\ 95 & -12 & 13 \\ -15 & 32 & 13 \end{vmatrix}$$

$$\text{area } \Delta ABC = \frac{132}{13} \text{ sq. units.}$$

13. Let $a > 0, b > 0$. Let e and ℓ respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and ℓ' respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14}\ell$ and $(e')^2 = \frac{11}{8}\ell'$, then the value of $77a + 44b$ is equal to

(A) 100 (B) 110

(C) 120 (D) 130

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $e = \sqrt{1 + \frac{b^2}{a^2}}, \ell = \frac{2b^2}{a}$

$$\text{Given } e^2 = \frac{11}{14}\ell$$

$$1 + \frac{b^2}{a^2} = \frac{11}{14} \cdot \frac{2b^2}{a}$$

$$\frac{a^2 + b^2}{a^2} = \frac{11}{7} \cdot \frac{b^2}{a} \quad \dots\dots(1)$$

Official Ans. by NTA (D)**Allen Ans. (D)****Sol.** DR'S normal of plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 18\hat{i} - \hat{j} + 7\hat{k}$$

 \therefore eqⁿ of plane

$18x - y + 7z = d$

It passes through (2, 3, -5)

$36 - 3 - 35 = d \quad \therefore d = -2$

 \therefore Eqⁿ of plane

$18x - y + 7z = -2$

$-18x + y - 7z = 2$

$\therefore a = -18, b = 1, c = -7, d = 2$

$a + 7b + c + 20d = -18 + 7 - 7 + 40 = 22$

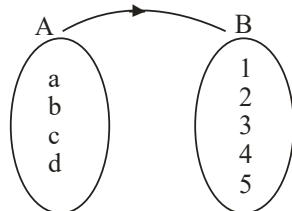
- 17.** The probability that a randomly chosen one-one function from the set {a, b, c, d} to the set {1, 2, 3, 4, 5} satisfies $f(a) + 2f(b) - f(c) = f(d)$ is :

(A) $\frac{1}{24}$

(B) $\frac{1}{40}$

(C) $\frac{1}{30}$

(D) $\frac{1}{20}$

Official Ans. by NTA (D)**Allen Ans. (D)****Sol.**

$n(s) = 5_{C_4} \times 4! = 120$

$f(a)$	$+ 2f(b)$	$=$	$f(c)$	$+ f(d)$
5	2×1	3	4	
4	2×2	3	5	
1	2×3	2	5	

$n(A) = 2! \times 3 = 6$

$\therefore P(A) = \frac{n(A)}{n(s)} = \frac{6}{120} = \frac{1}{20}$

- 18.** The value of $\lim_{n \rightarrow \infty} 6 \tan \left\{ \sum_{r=1}^n \tan^{-1} \left(\frac{1}{r^2 + 3r + 3} \right) \right\}$

 \therefore equal to

(A) 1 (B) 2

(C) 3 (D) 6

Official Ans. by NTA (C)**Allen Ans. (C)**

$$\text{Sol. } T_r = \tan^{-1} \left[\frac{(r+2)-(r+1)}{1+(r+2)(r+1)} \right]$$

$= \tan^{-1}(r+2) - \tan^{-1}(r+1)$

$T_1 = \tan^{-1} 3 - \tan^{-1} 2$

$T_2 = \tan^{-1} 4 - \tan^{-1} 3$

$T_n = \tan^{-1}(n+2) - \tan^{-1}(n+1)$

$S_n = \tan^{-1}(n+2) - \tan^{-1} 2 = \tan^{-1} \left(\frac{n+2-2}{1+2(n+2)} \right)$

$= \tan^{-1} \left(\frac{n}{2n+5} \right)$

$\lim_{n \rightarrow \infty} 6 \tan \left(\tan^{-1} \left(\frac{n}{2n+5} \right) \right)$

$= \lim_{n \rightarrow \infty} \frac{6n}{2n+5} = \frac{6}{2} = 3$

- 19.** Let \vec{a} be a vector which is perpendicular to the vector

$3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then

the projection of the vector \vec{a} on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is

(A) $\frac{1}{3}$ (B) 1

(C) $\frac{5}{3}$ (D) $\frac{7}{3}$

Official Ans. by NTA (C)**Allen Ans. (C)**

$$\text{Sol. } (\vec{a} \times (2\hat{i} + \hat{k})) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k} \right)$$

$$= (2\hat{i} - 13\hat{j} - 4\hat{k}) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k} \right)$$

$$-(6+2)\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -13 & -4 \\ 3 & \frac{1}{2} & 2 \end{vmatrix}$$

$$\vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$$

Projection of \vec{a} on vector $2\hat{i} + 2\hat{j} + \hat{k}$ is

$$\vec{a} \cdot \frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3} = \frac{5}{3}$$

20. If $\cot \alpha = 1$ and $\sec \beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$

and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and

the quadrant in which $\alpha + \beta$ lies, respectively are

(A) $-\frac{1}{7}$ and IVth quadrant

(B) 7 and Ist quadrant

(C) -7 and IVth quadrant

(D) $\frac{1}{7}$ and Ist quadrant

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $\cot \alpha = 1, \sec \beta = -\frac{5}{3}, \cos \beta = \frac{-3}{5}, \tan \beta = \frac{-4}{3}$

$$\tan(\alpha + \beta) = \frac{1 - \frac{4}{3}}{1 + \frac{4}{3} \times 1} = \frac{-1}{7}$$

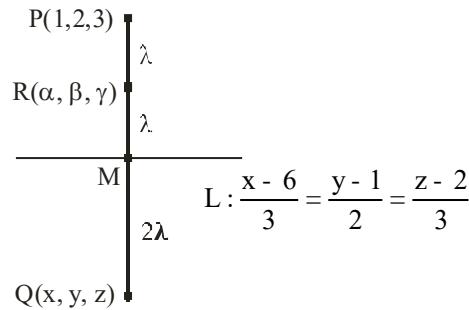
SECTION-B

1. Let the image of the point P(1, 2, 3) in the line L: $\frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be Q. let R(α, β, γ) be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22(\alpha + \beta + \gamma)$ is equal to

Official Ans. by NTA (125)

Allen Ans. (125)

Sol.



Let M be the mid-point of PQ

$$\therefore M = (3\lambda + 6, 2\lambda + 1, 3\lambda + 2)$$

$$\text{Now, } \overrightarrow{PM} = (3\lambda + 5)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 1)\hat{k}$$

$$\because \overrightarrow{PM} \perp (3\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\therefore 3(3\lambda + 5) + 2(2\lambda - 1) + 3(3\lambda - 1) = 0$$

$$\lambda = \frac{-5}{11}$$

$$\therefore M\left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11}\right)$$

Since R is mid-point of PM

$$22(\alpha + \beta + \gamma) = 125$$

2. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is

Official Ans. by NTA (0)

Allen Ans. (0)

Sol. $20 = \frac{\sum_{i=1}^7 |x_i - 62|^2}{7}$

$$\Rightarrow |x_1 - 62|^2 + |x_2 - 62|^2 + \dots + |x_7 - 62|^2 = 140$$

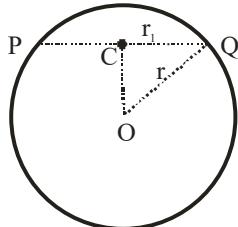
$$\text{If } x_1 = 49$$

$$|49 - 62|^2 = 169$$

then,

$|x_2 - 62|^2 + \dots + |x_7 - 62|^2 = \text{Negative Number}$
which is not possible, therefore, no student can fail.

3. If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$, then the value of r^2 is equal to
Official Ans. by NTA (10)

Allen Ans. (10)**Sol.**

PQ is diameter of circle

$$S: x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$$

$$C(\sqrt{2}, 3\sqrt{2}), O(2\sqrt{2}, 2\sqrt{2})$$

$$r_1 = \sqrt{6}$$

$$S_1: (x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$$

Now in ΔOCQ

$$|OC|^2 + |CQ|^2 = |OQ|^2$$

$$4 + 6 = r^2$$

$$r^2 = 10$$

4. If $\lim_{x \rightarrow 1} \frac{\sin(3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$, then the value of $(a - b)$ is equal to

Official Ans. by NTA (11)**Allen Ans. (11)**

$$\lim_{x \rightarrow 1} \frac{\sin(3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$$

For finite limit

$$a + b - 5 = 0 \quad \dots(1)$$

Apply L'H rule

$$\lim_{x \rightarrow 1} \frac{\cos(3x^2 - 4x + 1)(6x - 4) - 2x}{(6x^2 - 14x + a)} = -2$$

For finite limit

$$6 - 14 + a = 0$$

$$[a = 8]$$

$$\text{From (1)} \quad [b = -3]$$

$$\text{Now } (a - b) = 11$$

5. Let for $n = 1, 2, \dots, 50$, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the

$$\text{value of } \frac{1}{26} + \sum_{n=1}^{50} \left(S_n + \frac{2}{n+1} - n - 1 \right)$$

Official Ans. by NTA (41651)**Allen Ans. (41651)**

$$S_n = \frac{n^2}{1 - \frac{1}{(n+1)^2}} = \frac{n(n+1)^2}{(n+2)}$$

$$S_n = \frac{n(n^2 + 2n + 1)}{(n+2)}$$

$$S_n = \frac{n[n(n+2)+1]}{(n+2)}$$

$$S_n = n \left[n + \frac{1}{n+2} \right]$$

$$S_n = n^2 + 1 - \frac{2}{(n+2)}$$

$$S_n = n^2 + 1 - \frac{2}{(n+2)}$$

$$\text{Now } \frac{1}{26} + \sum_{n=1}^{50} \left[(n^2 - n) - 2 \left(\frac{1}{n+2} - \frac{1}{n+1} \right) \right]$$

$$= \frac{1}{26} + \left[\frac{50 \times 51 \times 101}{6} - \frac{50 \times 51}{2} - 2 \left(\frac{1}{52} - \frac{1}{2} \right) \right]$$

$$= 41651$$

6. If the system of linear equations

$$2x - 3y = \gamma + 5,$$

$\alpha x + 5y = \beta + 1$, where $\alpha, \beta, \gamma \in \mathbf{R}$ has infinitely many solutions, then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to

Official Ans. by NTA (58)**Allen Ans. (58)**

Sol. $2x - 3y = \gamma + 5$

$$\alpha x + 5y = \beta + 1$$

Infinite many solution

$$\frac{\alpha}{2} = \frac{5}{-3} = \frac{\beta+1}{\gamma+5}$$

$$\alpha = \frac{-10}{3}, \quad 5\gamma + 25 = -3\beta - 3$$

$$9\alpha = -30, \quad 3\beta + 5\gamma = -28$$

$$\text{Now, } 9\alpha + 3\beta + 5\gamma = -58$$

$$|9\alpha + 3\beta + 5\gamma| = 58$$

7. Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$.

Then, the number of elements in the set

$$\{n \in \{1, 2, \dots, 100\} : A^n = A\}$$

Official Ans. by NTA (25)

Allen Ans. (25)

Sol. $A = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix} \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{4n+1} = A$$

$$n = 1, 5, 9, \dots, 97$$

\Rightarrow total elements in the set is 25.

8. Sum of squares of modulus of all the complex numbers z satisfying $\bar{z} = iz^2 + z^2 - z$ is equal to

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $z + \bar{z} = iz^2 + z^2$

Consider $z = x + iy$

$$2x = (i+1)(x^2 - y^2 + 2xyi)$$

$$\Rightarrow 2x = x^2 - y^2 - 2xy \text{ and } x^2 - y^2 + 2xy = 0$$

$$\Rightarrow 2x = -4xy$$

$$\Rightarrow x = 0 \text{ or } y = \frac{-1}{2}$$

Case 1 : $x = 0 \Rightarrow y = 0$ here $z = 0$

Case 2 : $y = \frac{-1}{2}$

$$\Rightarrow 4x^2 - 4x - 1 = 0$$

$$(2x-1)^2 = 2$$

$$2x-1 = \pm\sqrt{2}$$

$$x = \frac{1 \pm \sqrt{2}}{2}$$

$$\text{Here } z = \frac{1+\sqrt{2}}{2} - \frac{i}{2} \text{ or } z = \frac{1-\sqrt{2}}{2} - \frac{i}{2}$$

Sum of squares of modulus of z

$$= 0 + \frac{(1+\sqrt{2})^2 + 1}{4} + \frac{(1-\sqrt{2})^2 + 1}{4} = \frac{8}{4} = 2$$

9. Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : S \times S \rightarrow S : f \text{ is onto and } f(a, b) = f(b, a) \geq a \forall (a, b) \in S \times S\}$ is

Official Ans. by NTA (37)

Allen Ans. (37)

- Sol.** $(1, 1), (1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 4)$ – all have one choice for image.

$(2, 1), (1, 2), (2, 2)$ – all have three choices for image

$(3, 2), (2, 3), (3, 1), (1, 3), (3, 3)$ – all have two choices for image.

So the total functions = $3 \times 3 \times 2 \times 2 \times 2 = 72$

Case 1 : None of the pre-images have 3 as image

Total functions = $2 \times 2 \times 1 \times 1 \times 1 = 4$

Case 2 : None of the pre-images have 2 as image

Total functions = $2 \times 2 \times 2 \times 2 \times 2 = 32$

Case 3 : None of the pre-images have either 3 or 2 as image

Total functions = $1 \times 1 \times 1 \times 1 \times 1 = 1$

\therefore Total onto functions = $72 - 4 - 32 + 1 = 37$

10. The maximum number of compound propositions, out of $p \vee r \vee s$, $p \vee r \vee \sim s$, $p \vee \sim q \vee s$,
 $\sim p \vee \sim r \vee s$, $\sim p \vee \sim r \vee \sim s$, $\sim p \vee q \vee \sim s$,
 $q \vee r \vee \sim s$, $q \vee \sim r \vee \sim s$, $\sim p \vee \sim q \vee \sim s$

that can be made simultaneously true by an assignment of the truth values to p, q, r and s, is equal to

Official Ans. by NTA (9)

Allen Ans. (9)

Sol. If we take

p	q	r	s
F	F	T	F

The truth value of all the propositions will be true.