## Mallem D \| G \| T A L JEE-MAIN - JUNE, 2022

## Mathematics

Test Pattern : JEE-MAIN
Maximum Marks : 120

## Topic Covered: FULL SYLLABUS

## Important instruction:

1. Use Blue / Black Ball point pen only.
2. There are three sections of equal weightage in the question paper Physics, Chemistry and Mathematics having 30 questions in each subject. Each paper have 2 sections $A$ and $B$.
3. You are awarded +4 marks for each correct answer and -1 marks for each incorrect answer.
4. Use of calculator and other electronic devices is not allowed during the exam.
5. No extra sheets will be provided for any kind of work.
```
Name of the Candidate (in Capitals)
```

Father's Name (in Capitals)
Form Number : in figures
: in words
Centre of Examination (in Capitals):
Candidate's Signature: $\qquad$ Invigilator's Signature : $\qquad$

## Rough Space

## YOUR TARGET IS TO SECURE GOOD RANK IN JEE-MAIN

## FINAL JEE-MAIN EXAMINATION - JUNE, 2022

(Held On Tuesday 28 ${ }^{\text {th }}$ June, 2022)
TIME: 3:00 PM to 06:00 PM

## MATHEMATICS

## SECTION-A

1. Let $\mathrm{R}_{1}=\{(\mathrm{a}, \mathrm{b}) \in \mathrm{N} \times \mathrm{N}:|\mathrm{a}-\mathrm{b}| \leq 13\}$ and

$$
R_{2}=\{(a, b) \in N \times N:|a-b| \neq 13\} \text {. Then on } N:
$$

(A) Both $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ are equivalence relations
(B) Neither $R_{1}$ nor $R_{2}$ is an equivalence relation
(C) $R_{1}$ is an equivalence relation but $R_{2}$ is not
(D) $\mathrm{R}_{2}$ is an equivalence relation but $\mathrm{R}_{1}$ is not

## Official Ans. by NTA (B)

Allen Ans. (B)
Sol. $\mathrm{R}_{1}=\{(\mathrm{a}, \mathrm{b}) \in \mathrm{N} \times \mathrm{N}:|\mathrm{a}-\mathrm{b}| \leq 13\}$ $R_{2}=\{(a, b) \in N \times N:|a-b| \neq 13\}$.
For $\mathrm{R}_{1}$ :
i) Reflexive relation

$$
(a, a) \in N \times N:|a-a| \leq 13
$$

ii) Symmetric relation

$$
(\mathrm{a}, \mathrm{~b}) \in \mathrm{R}_{1},(\mathrm{~b}, \mathrm{a}) \in \mathrm{R}_{1}:|\mathrm{b}-\mathrm{a}| \leq 13
$$

iii) Transitive relation
$(a, b) \in R_{1},(b, c) \in R_{1},(a, c) \in R_{1}:$
$(1,3) \in \mathrm{R}_{1},(3,16) \in \mathrm{R}_{1}$, but $(1,16) \notin \mathrm{R}_{1}$
For $\mathrm{R}_{2}$ :
i) Reflexive relation
$(a, a) \in N \times N:|a-a| \neq 13$
ii) Symmetric relation
$(b, a) \in N \times N:|b-a| \neq 13$
iii) Transitive relation
$(\mathrm{a}, \mathrm{b}) \in \mathrm{R}_{2},(\mathrm{~b}, \mathrm{c}) \in \mathrm{R}_{2},(\mathrm{a}, \mathrm{c}) \in \mathrm{R}_{2}$
$(1,3) \in \mathrm{R}_{2},(3,14) \in \mathrm{R}_{2}$, but $(1,14) \notin \mathrm{R}_{2}$
2. Let $f(x)$ be a quadratic polynomial such that $f(-2)$ $+f(3)=0$. If one of the roots of $f(x)=0$ is -1 , then the sum of the roots of $f(x)=0$ is equal to :
(A) $\frac{11}{3}$
(B) $\frac{7}{3}$
(C) $\frac{13}{3}$
(D) $\frac{14}{3}$

Official Ans. by NTA (A)
Allen Ans. (A)

## TEST PAPER WITH SOLUTION

Sol. $f(-2)+f(3)=0$
$\mathrm{f}(\mathrm{x})=(\mathrm{x}+1)(\mathrm{ax}+\mathrm{b})$
$f(-2)+f(3)=-1(-2 a+b)+4(3 a+b)=0$
$2 a-b+12 a+4 b=0$
$14 a+3 b=0$
$\frac{-\mathrm{b}}{\mathrm{a}}=\frac{14}{3}$
Sum of roots $=\left(-1+\frac{-\mathrm{b}}{\mathrm{a}}\right)=-1+\frac{14}{3}=\frac{11}{3}$
3. The number of ways to distribute 30 identical candies among four children $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}$ and $\mathrm{C}_{4}$ so that $\mathrm{C}_{2}$ receives atleast 4 and atmost 7 candies, $\mathrm{C}_{3}$ receives atleast 2 and atmost 6 candies, is equal to
(A) 205
(B) 615
(C) 510
(D) 430

Official Ans. by NTA (D)
Allen Ans. (D)
Sol. $t_{1}+t_{2}+t_{3}+t_{4}=30$
Coefficient of $x^{30}$ in $\left(1+x+x^{2}+\ldots+x^{30}\right)^{2}$
$\left(x^{4}+x^{5}+x^{6}+x^{7}\right)\left(x^{2}+x^{3}+x^{4}+x^{5}+x^{6}\right)$
$x^{6}\left(\frac{1-x^{31}}{1-x}\right)^{2}\left(1+x+x^{2}+x^{3}\right)\left(1+x+x^{2}+x^{3}+x^{4}\right)$
$x^{6}\left(1-x^{31}\right)^{2}\left(1-x^{4}\right)\left(1-x^{5}\right)(1-x)^{4}$
$x^{6}\left(1-x^{4}-x^{5}+x^{9}\right)\left(1+x^{62}-2 x^{31}(1-x)^{-4}\right)$
$x^{6}\left(1-x^{4}-x^{5}+x^{9}\right)(1-x)^{-4}$
Coefficient of $x^{n}$ in $(1-x)^{-r}$ is ${ }^{n+r-1} C_{r-1}$
$\Rightarrow{ }^{27} \mathrm{C}_{3}-{ }^{23} \mathrm{C}_{3}-{ }^{22} \mathrm{C}_{3}+{ }^{18} \mathrm{C}_{3}$
2925-1771-1540+816
$=430$
OR
$\mathrm{x}_{2} \in[4,7], \mathrm{x}_{3} \in[2,6]$
$\Rightarrow \mathrm{t}_{1}+\mathrm{t}_{2}+\mathrm{t}_{3}+\mathrm{t}_{4}=24$
total ways $=$
${ }^{24+4-1} \mathrm{C}_{4-1}-{ }^{20+4-1} \mathrm{C}_{4-1}-{ }^{19+4-1} \mathrm{C}_{4-1}+{ }^{15+4-1} \mathrm{C}_{4-1}$
$={ }^{27} \mathrm{C}_{3}-{ }^{23} \mathrm{C}_{3}-{ }^{22} \mathrm{C}_{3}+{ }^{18} \mathrm{C}_{3}=430$
4. The term independent of $x$ in the expression of $\left(1-x^{2}+3 x^{3}\right)\left(\frac{5}{2} x^{3}-\frac{1}{5 x^{2}}\right)^{11}, x \neq 0$ is
(A) $\frac{7}{40}$
(B) $\frac{33}{200}$
(C) $\frac{39}{200}$
(D) $\frac{11}{50}$

Official Ans. by NTA (B)
Allen Ans. (B)
Sol. $\left(1-x^{2}+3 x^{3}\right)\left(\frac{5}{2} x^{3}-\frac{1}{5 x^{2}}\right)^{11}$
General term of $\left(\frac{5}{2} x^{3}-\frac{1}{5 x^{2}}\right)^{11}$ is
${ }^{11} C_{r}\left(\frac{5}{2} x^{3}\right)^{11-r}\left(-\frac{1}{5 x^{2}}\right)^{r}$
General term is ${ }^{11} C_{r}\left(\frac{5}{2}\right)^{11-\mathrm{r}}\left(-\frac{1}{5}\right)^{\mathrm{r}} \mathrm{x}^{33-5 \mathrm{r}}$
Now, term independent of $x$
$1 \times$ coefficient of $x^{0}$ in $\left(\frac{5}{2} x^{3}-\frac{1}{5 x^{2}}\right)^{11}$
$-1 \times$ coefficient of $x^{-2}$ in $\left(\frac{5}{2} x^{3}-\frac{1}{5 x^{2}}\right)^{11}+$
$3 \times$ coefficient of $x^{-3}$ in $\left(\frac{5}{2} x^{3}-\frac{1}{5 x^{2}}\right)^{11}$
for coefficient of $x^{0} \quad 33-5 r=0$ not possible
for coefficient of $x^{-2}$
$33-5 r=-2$
$35=5 \mathrm{r} \Rightarrow \mathrm{r}=7$
for coefficient of $\mathrm{x}^{-3}$
$33-5 r=-3$
$36=5 \mathrm{r}$ not possible
So term independent of x is
$(-1){ }^{11} \mathrm{C}_{7}\left(\frac{5}{2}\right)^{4}\left(-\frac{1}{5}\right)^{7}=\frac{33}{200}$
5. If $n$ arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1: 7$ and $a+n=33$, then the value of $n$ is
(A) 21
(B) 22
(C) 23
(D) 24

Official Ans. by NTA (C)
Allen Ans. (C)

Sol. $\mathrm{d}=\frac{100-\mathrm{a}}{\mathrm{n}+1}$
$A_{1}=a+d$
$A_{n}=100-d$
$\Rightarrow \frac{\mathrm{A}_{1}}{\mathrm{~A}_{\mathrm{n}}}=\frac{1}{7} \Rightarrow \frac{\mathrm{a}+\mathrm{d}}{100-\mathrm{d}}=\frac{1}{7}$
$\Rightarrow 7 \mathrm{a}+8 \mathrm{~d}=100$
$\Rightarrow 7 \mathrm{a}+8\left(\frac{100-\mathrm{a}}{\mathrm{n}+1}\right)=100$
$\because a+n=33$
Now, by Eq. (1) and (2)
$7 n^{2}-132 n-667=0$
$\mathrm{n}=23$ and $\mathrm{n}=\frac{-29}{7}$ reject.
6. Let $\mathrm{f}, \mathrm{g}: \mathbf{R} \rightarrow \mathbf{R}$ be functions defined by
$f(x)=\left\{\begin{array}{ll}{[x]} & , \quad x<0 \\ |1-x| & , \quad x \geq 0\end{array}\right.$ and
$g(x)= \begin{cases}e^{x}-x & , \\ (x-1)^{2}-1 & , \\ x \geq 0\end{cases}$
where $[\mathrm{x}$ ] denote the greatest integer less than or equal to $x$. Then, the function fog is discontinuous at exactly :
(A) one point
(B) two points
(C) three points
(D) four points

Official Ans. by NTA (B)
Allen Ans. (B)
Sol. Check continuity at $\mathrm{x}=0$ and also check continuity at those x where $\mathrm{g}(\mathrm{x})=0$
$\mathrm{g}(\mathrm{x})=0$ at $\mathrm{x}=0,2$
$\operatorname{fog}\left(0^{+}\right)=-1$
$\operatorname{fog}(0)=0$
Hence, discontinuous at $\mathrm{x}=0$
fog $\left(2^{+}\right)=1$
$\operatorname{fog}\left(2^{-}\right)=-1$
Hence, discontinuous at $\mathrm{x}=2$
7. Let $\mathrm{f}: \mathbf{R} \rightarrow \mathbf{R}$ be a differentiable function such that $\mathrm{f}\left(\frac{\pi}{4}\right)=\sqrt{2}, \mathrm{f}\left(\frac{\pi}{2}\right)=0$ and $\mathrm{f}^{\prime}\left(\frac{\pi}{2}\right)=1$ and let $g(x)=\int_{x}^{\pi / 4}\left(f^{\prime}(t) \sec t+\tan t \sec t f(t)\right) d t$ for $\mathrm{x} \in\left[\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then $\lim _{\mathrm{x} \rightarrow\left(\frac{\pi}{2}\right)^{-}} \mathrm{g}(\mathrm{x})$ is equal to
(A) 2
(B) 3
(C) 4
(D) -3

Official Ans. by NTA (B)
Allen Ans. (B)
Sol. $\quad g(x)=\int_{x}^{\pi / 4}\left(f^{\prime}(t) \sec t+\tan t \sec t f(t)\right) d t$
$g(x)=\int_{x}^{\pi / 4} d(f(t) \cdot \sec t)=\left.f(t) \sec t\right|_{x} ^{\pi / 4}$
$g(x)=f\left(\frac{\pi}{4}\right) \sec \frac{\pi}{4}-f(x) \cdot \sec x$
$g(x)=2-f(x) \sec x=2-\left(\frac{f(x)}{\cos x}\right)$
$\lim _{x \rightarrow\left(\frac{\pi}{2}\right)^{-}} g(x)=2-\lim _{x \rightarrow\left(\frac{\pi}{2}\right)^{-}}\left(\frac{f(x)}{\cos x}\right)$
Using L'Hopital Rule
$=2-\lim _{x \rightarrow\left(\frac{\pi}{2}\right)^{-}} \frac{f^{\prime}(x)}{(-\sin x)}$
$=2+\frac{\mathrm{f}^{\prime}\left(\frac{\pi}{2}\right)}{\sin \frac{\pi}{2}}=2+\frac{1}{1}=3$
8. Let $\mathrm{f}: \mathbf{R} \rightarrow \mathbf{R}$ be continuous function satisfying $\mathrm{f}(\mathrm{x})+\mathrm{f}(\mathrm{x}+\mathrm{k})=\mathrm{n}$, for all $\mathrm{x} \in \mathbf{R}$ where $\mathrm{k}>0$ and n
is a positive integer. If $I_{1}=\int_{0}^{4 n k} f(x) d x$ and $I_{2}=\int_{-k}^{3 k} f(x) d x$, then
(A) $\mathrm{I}_{1}+2 \mathrm{I}_{2}=4 \mathrm{nk}$
(B) $\mathrm{I}_{1}+2 \mathrm{I}_{2}=2 \mathrm{nk}$
(C) $\mathrm{I}_{1}+\mathrm{nI}_{2}=4 \mathrm{n}^{2} \mathrm{k}$
(D) $\mathrm{I}_{1}+\mathrm{nI}_{2}=6 \mathrm{n}^{2} \mathrm{k}$

## Official Ans. by NTA (C)

Allen Ans. (C)
Sol. $\quad \mathrm{f}(\mathrm{x})+\mathrm{f}(\mathrm{x}+\mathrm{k})=\mathrm{n}$
$\Rightarrow \mathrm{f}(\mathrm{x})=\mathrm{f}(\mathrm{x}+2 \mathrm{k})$
$f(x)$ is periodic with period $2 k$
$I_{1}=\int_{0}^{4 n k} f(x) d x=2 n \int_{0}^{2 k} f(x) d x$
$I_{2}=\int_{-k}^{3 k} f(x) d x=2 \int_{0}^{2 k} f(x) d x$
Now,
$\mathrm{f}(\mathrm{x})+\mathrm{f}(\mathrm{x}+\mathrm{k})=\mathrm{n}$
$\Rightarrow \int_{0}^{k} f(x) d x+\int_{0}^{k} f(x+k) d x=n k$
$\Rightarrow \int_{0}^{k} f(x) d x+\int_{k}^{2 k} f(x) d x=n k$
$\Rightarrow \int_{0}^{2 \mathrm{k}} \mathrm{f}(\mathrm{x}) \mathrm{dx}=\mathrm{nk}$
$\Rightarrow \mathrm{I}_{1}=2 \mathrm{n}^{2} \mathrm{k}, \mathrm{I}_{2}=2 \mathrm{nk}$
$\Rightarrow \mathrm{I}_{1}+\mathrm{nI}_{2}=4 \mathrm{n}^{2} \mathrm{k}$
9. The area of the bounded region enclosed by the curve $\mathrm{y}=3-\left|\mathrm{x}-\frac{1}{2}\right|-|\mathrm{x}+1|$ and the x -axis is
(A) $\frac{9}{4}$
(B) $\frac{45}{16}$
(C) $\frac{27}{8}$
(D) $\frac{63}{16}$

Official Ans. by NTA (C)
Allen Ans. (C)
$\int 3+(x+1)+\left(x-\frac{1}{2}\right), \quad x<-1$
Sol. $y= \begin{cases}3-(x+1)+\left(x-\frac{1}{2}\right), & -1 \leq x<\frac{1}{2} \\ 3-(x+1)-\left(x-\frac{1}{2}\right), & \frac{1}{2} \leq x\end{cases}$

$$
y= \begin{cases}\frac{7}{2}+2 x, & x<-1 \\ \frac{3}{2}, & -1 \leq x<\frac{1}{2} \\ \frac{5}{2}-2 x, & \frac{1}{2} \leq x\end{cases}
$$



Area bounded $=$ ar $\mathrm{ABF}+$ ar $\mathrm{BCEF}+$ ar CDE
$=\frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)+\frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{3}{2}\right)$
$=\frac{27}{8}$ sq. units.
10. Let $x=x(y)$ be the solution of the differential equation $2 \mathrm{ye}^{\mathrm{x} / \mathrm{y}^{2}} \mathrm{dx}+\left(\mathrm{y}^{2}-4 \mathrm{xe}^{\mathrm{x} / \mathrm{y}^{2}}\right) \mathrm{dy}=0$ such that $\mathrm{x}(1)=0$. Then, $\mathrm{x}(\mathrm{e})$ is equal to
(A) $e \log _{e}(2)$
(B) $-\mathrm{e} \log _{\mathrm{e}}(2)$
(C) $e^{2} \log _{e}(2)$
(D) $-\mathrm{e}^{2} \log _{\mathrm{e}}(2)$

Official Ans. by NTA (D)
Allen Ans. (D)
Sol. $2 y^{x / y^{2}} d x+\left(y^{2}-4 x e^{x / y^{2}}\right) d y=0$
$2 \mathrm{e}^{\mathrm{x} / \mathrm{y}^{2}}[\mathrm{ydx}-2 \mathrm{xdy}]+\mathrm{y}^{2} \mathrm{dy}=0$
$2 e^{x / y^{2}}\left[\frac{y^{2} d x-x \cdot(2 y) d y}{y}\right]+y^{2} d y=0$
Divide by y ${ }^{3}$
$2 e^{x / y^{2}}\left[\frac{y^{2} d x-x \cdot(2 y) d y}{y^{4}}\right]+\frac{1}{y} d y=0$
$2 e^{x / y^{2}} d\left(\frac{x}{y^{2}}\right)+\frac{1}{y} d y=0$
Integrating
$\int 2 e^{x / y^{2}} d\left(\frac{x}{y^{2}}\right)+\int \frac{1}{y} d y=0$
$2 e^{x / y^{2}}+$ lny $+c=0$
$(0,1)$ lies on it.
$2 \mathrm{e}^{0}+\ln 1+\mathrm{c}=0 \Rightarrow \mathrm{c}=-2$
Required curve : $2 \mathrm{e}^{\mathrm{x} / \mathrm{y}^{2}}+\ell$ ny $-2=0$
For $x(e)$
$2 \mathrm{e}^{\mathrm{x} / \mathrm{e}^{2}}+\ell \mathrm{ne}-2=0 \Rightarrow \mathrm{x}=-\mathrm{e}^{2} \log _{\mathrm{e}} 2$
11. Let the slope of the tangent to a curve $y=f(x)$ at $(\mathrm{x}, \mathrm{y})$ be given by $2 \tan \mathrm{x}(\cos \mathrm{x}-\mathrm{y})$. if the curve passes through the point $(\pi / 4,0)$, then the value of $\int_{0}^{\pi / 2} y d x$ is equal to
(A) $(2-\sqrt{2})+\frac{\pi}{\sqrt{2}}$
(B) $2-\frac{\pi}{\sqrt{2}}$
(C) $(2+\sqrt{2})+\frac{\pi}{\sqrt{2}}$
(D) $2+\frac{\pi}{\sqrt{2}}$

Official Ans. by NTA (B)

## Allen Ans. (B)

Sol. $\frac{d y}{d x}=2 \tan x \cos x-2 \tan x \cdot y$
$\frac{d y}{d x}+(2 \tan x) y=2 \sin x$
Integrating factor $=e^{\int 2 \tan x d x}=\frac{1}{\cos ^{2} x}$
$y\left(\frac{1}{\cos ^{2} x}\right)=\int \frac{2 \sin x}{\cos ^{2} x} d x$
$y \sec ^{2} x=\frac{2}{\cos x}+C$
$y=2 \cos x+C \cos ^{2} x$
Passes through $\left(\frac{\pi}{4}, 0\right)$
$0=\sqrt{2}+\frac{C}{2} \Rightarrow \mathrm{C}=-2 \sqrt{2}$
$f(x)=2 \cos x-2 \sqrt{2} \cos ^{2} x:$ Required curve
$\int_{0}^{\pi / 2} y d x=2 \int_{0}^{\pi / 2} \cos x d x-2 \sqrt{2} \int_{0}^{\pi / 2} \cos ^{2} x d x$
$=[2 \sin x]_{0}^{\pi / 2}-2 \sqrt{2}\left[\frac{x}{2}+\frac{\sin 2 x}{4}\right]_{0}^{\pi / 2}$
$=2-\frac{\pi}{\sqrt{2}}$
12. Let a triangle be bounded by the lines $\mathrm{L}_{1}: 2 x+5 y=10$; $L_{2}:-4 x+3 y=12$ and the line $L_{3}$, which passes through the point $\mathrm{P}(2,3)$, intersect $\mathrm{L}_{2}$ at A and $\mathrm{L}_{1}$ at $B$. If the point $P$ divides the line-segment $A B$, internally in the ratio $1: 3$, then the area of the triangle is equal to
(A) $\frac{110}{13}$
(B) $\frac{132}{13}$
(C) $\frac{142}{13}$
(D) $\frac{151}{13}$

Official Ans. by NTA (B)

## Allen Ans. (B)

Sol. Points A lies on $L_{2}$
$\mathrm{A}\left(\alpha, 4+\frac{4}{3} \alpha\right)$
Points B lies on $\mathrm{L}_{1}$
$\mathrm{B}\left(\beta, 2-\frac{2}{5} \beta\right)$
Points P divides AB internally in the ratio $1: 3$
$\Rightarrow P(2,3)=P\left(\frac{3 \alpha+\beta}{4}, \frac{3\left(4+\frac{4}{3} \alpha\right)+1\left(2-\frac{2}{5} \beta\right)}{4}\right)$
$\Rightarrow \alpha=\frac{3}{13}, \beta=\frac{95}{13}$

Point $\mathrm{A}\left(\frac{3}{13}, \frac{56}{13}\right), \mathrm{B}\left(\frac{95}{13},-\frac{12}{13}\right)$
Vertex C of triangle is the point of intersection of $\mathrm{L}_{1} \& \mathrm{~L}_{2}$
$\Rightarrow \mathrm{C}\left(-\frac{15}{13}, \frac{32}{13}\right)$
$\operatorname{area} \Delta \mathrm{ABC}=\frac{1}{2}\left\|\begin{array}{|lcc}\frac{3}{13} & \frac{56}{13} & 1\end{array}\right\|$

$$
=\frac{1}{2 \times 13^{3}}\left\|\begin{array}{ccc}
3 & 56 & 13 \\
95 & -12 & 13 \\
-15 & 32 & 13
\end{array}\right\|
$$

area $\triangle \mathrm{ABC}=\frac{132}{13}$ sq. units.
13. Let $\mathrm{a}>0, \mathrm{~b}>0$. Let e and $\ell$ respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{\mathrm{x}^{2}}{\mathrm{a}^{2}}-\frac{\mathrm{y}^{2}}{\mathrm{~b}^{2}}=1$. Let $\mathrm{e}^{\prime}$ and $\ell^{\prime}$ respectively the eccentricity and length of the latus rectum of its conjugate hyperbola. If $\mathrm{e}^{2}=\frac{11}{14} \ell$ and $\left(\mathrm{e}^{\prime}\right)^{2}=\frac{11}{8} \ell^{\prime}$, then the value of $77 a+44 b$ is equal to
(A) 100
(B) 110
(C) 120
(D) 130

Official Ans. by NTA (D)
Allen Ans. (D)
Sol. $\mathrm{e}=\sqrt{1+\frac{\mathrm{b}^{2}}{\mathrm{a}^{2}}}, \quad \ell=\frac{2 \mathrm{~b}^{2}}{\mathrm{a}}$
Given $\mathrm{e}^{2}=\frac{11}{14} \ell$
$1+\frac{\mathrm{b}^{2}}{\mathrm{a}^{2}}=\frac{11}{14} \cdot \frac{2 \mathrm{~b}^{2}}{\mathrm{a}}$
$\frac{\mathrm{a}^{2}+\mathrm{b}^{2}}{\mathrm{a}^{2}}=\frac{11}{7} \cdot \frac{\mathrm{~b}^{2}}{\mathrm{a}}$

Also $\mathrm{e}^{\prime}=\sqrt{1+\frac{\mathrm{a}^{2}}{\mathrm{~b}^{2}}}, \ell^{\prime}=\frac{2 \mathrm{a}^{2}}{\mathrm{~b}}$
Given $\left(\mathrm{e}^{\prime}\right)^{2}=\frac{11}{8} \ell^{\prime}$
$1+\frac{\mathrm{a}^{2}}{\mathrm{~b}^{2}}=\frac{11}{8} \cdot \frac{2 \mathrm{a}^{2}}{\mathrm{~b}}$
$\frac{\mathrm{a}^{2}+\mathrm{b}^{2}}{\mathrm{~b}^{2}}=\frac{11}{4} \cdot \frac{\mathrm{a}^{2}}{\mathrm{~b}}$
New (1) $\div(2)$
$\frac{\mathrm{b}^{2}}{\mathrm{a}^{2}}=\frac{4}{7} \cdot \frac{\mathrm{~b}^{3}}{\mathrm{a}^{3}}$
$\therefore 7 a=4 b$
From (2)
$\frac{\frac{16 b^{2}}{49}+b^{2}}{b^{2}}=\frac{11}{4} \cdot \frac{16 b^{2}}{49 b}$
$\frac{65}{49}=\frac{11}{4} \cdot \frac{16}{49} . b$
$\therefore \mathrm{b}=\frac{4 \times 65}{11 \times 16}$
We have to find value of
$77 a+44 b$
$11(7 a+4 b)=11(4 b+4 b)=11 \times 8 b$
$\therefore$ Value of $11 \times 8 \mathrm{~b}=11 \times 8 \times \frac{4 \times 65}{16 \times 11}=130$
14. Let $\vec{a}=\alpha \hat{i}+2 \hat{j}-\hat{k}$ and $\vec{b}=-2 \hat{i}+\alpha \hat{j}+\hat{k}$, where $\alpha \in \mathbf{R}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15\left(\alpha^{2}+4\right)}$, then the value of $2|\overrightarrow{\mathrm{a}}|^{2}+(\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{b}})|\overrightarrow{\mathrm{b}}|^{2}$ is equal to
(A) 10
(B) 7
(C) 9
(D) 14

Official Ans. by NTA (D)
Allen Ans. (D)
Sol. $\quad \vec{a}=\alpha \hat{i}+2 \hat{j}-\hat{k}, \vec{b}=-2 \hat{i}+\alpha \hat{j}+\hat{k}$, area of parallelogram $=|\hat{a} \times \hat{b}|$
$|\hat{\mathrm{a}} \times \hat{\mathrm{b}}|=\sqrt{(\alpha+2)^{2}+(\alpha-2)^{2}+\left(\alpha^{2}+4\right)^{2}}$
Given $|\hat{\mathrm{a}} \times \hat{\mathrm{b}}|=\sqrt{15\left(\alpha^{2}+4\right)}$
$2\left(\alpha^{2}+4\right)+\left(\alpha^{2}+4\right)^{2}=15\left(\alpha^{2}+4\right)$
$\left(\alpha^{2}+4\right)^{2}=13\left(\alpha^{2}+4\right)$
$\Rightarrow \alpha^{2}+4=13 \therefore \alpha^{2}=9$
$2|\vec{a}|^{2}+(\vec{a} \cdot \vec{b})|\vec{b}|^{2}$
$|\overrightarrow{\mathrm{a}}|^{2}=\alpha^{2}+4+1=\alpha^{2}+5$
$|\overrightarrow{\mathrm{b}}|^{2}=4+\alpha^{2}+1=\alpha^{2}+5$
$\vec{a} \cdot \vec{b}=-2 \alpha+2 \alpha-1=-1$
$\therefore 2|\vec{a}|^{2}+(\vec{a} \cdot \vec{b})|\vec{b}|^{2}$
$2\left(\alpha^{2}+5\right)-1\left(\alpha^{2}+5\right)=\alpha^{2}+5=14$
15. If vertex of a parabola is $(2,-1)$ and the equation of its directrix is $4 x-3 y=21$, then the length of its latus rectum is
(A) 2
(B) 8
(C) 12
(D) 16

Official Ans. by NTA (B)
Allen Ans. (B)


Sol.

$$
4 x-3 y=21
$$

$a=\frac{|8+3-21|}{5}=\frac{10}{5}=2$
$\therefore$ latus rectum $=4 a=8$
16. Let the plane $a x+b y+c z=d$ pass through $(2,3,-5)$ and is perpendicular to the planes $2 \mathrm{x}+\mathrm{y}-5 \mathrm{z}=10$ and $3 x+5 y-7 z=12$.
If $a, b, c, d$ are integers $d>0$ and $\operatorname{gcd}(|a|,|b|,|c|, d)$ $=1$, then the value of $\mathrm{a}+7 \mathrm{~b}+\mathrm{c}+20 \mathrm{~d}$ is equal to
(A) 18
(B) 20
(C) 24
(D) 22

Official Ans. by NTA (D)
Allen Ans. (D)
Sol. DR'S normal of plane
$\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -5 \\ 3 & 5 & -7\end{array}\right|=18 \hat{i}-\hat{j}+7 \hat{k}$
$\therefore \mathrm{eq}^{\mathrm{n}}$ of plane
$18 \mathrm{x}-\mathrm{y}+7 \mathrm{z}=\mathrm{d}$
It passes through $(2,3,-5)$
$36-3-35=d$ $\therefore \mathrm{d}=-2$
$\therefore \mathrm{Eq}^{\mathrm{n}}$ of plane
$18 x-y+7 z=-2$
$-18 x+y-7 z=2$
$\therefore \mathrm{a}=-18, \mathrm{~b}=1, \mathrm{c}=-7, \mathrm{~d}=2$
$a+7 b+c+20 d=-18+7-7+40=22$
17. The probability that a randomly chosen one-one function from the set $\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$ to the set $\{1,2,3,4,5\}$ satisfies $f(a)+2 f(b)-f(c)=f(d)$ is :
(A) $\frac{1}{24}$
(B) $\frac{1}{40}$
(C) $\frac{1}{30}$
(D) $\frac{1}{20}$

Official Ans. by NTA (D)
Allen Ans. (D)

Sol.

$\mathrm{n}(\mathrm{s})=5_{\mathrm{C}_{4}} \times 4!=120$

| $\mathrm{f}(\mathrm{a})$ | $+2 \mathrm{f}(\mathrm{b})=\mathrm{f}(\mathrm{c})$ | $+\mathrm{f}(\mathrm{d})$ |  |
| :--- | :--- | :--- | :--- |
| 5 | $2 \times 1$ | 3 | 4 |
| 4 | $2 \times 2$ | 3 | 5 |
| 1 | $2 \times 3$ | 2 | 5 |

$n(A)=21 \times 3=6$
$\therefore \mathrm{P}(\mathrm{A})=\frac{\mathrm{n}(\mathrm{A})}{\mathrm{n}(\mathrm{s})}=\frac{6}{120}=\frac{1}{20}$
18. The value of $\lim _{\mathrm{n} \rightarrow \infty} 6 \tan \left\{\sum_{\mathrm{r}=1}^{\mathrm{n}} \tan ^{-1}\left(\frac{1}{\mathrm{r}^{2}+3 \mathrm{r}+3}\right)\right\}$ is equal to
(A) 1
(B) 2
(C) 3
(D) 6

Official Ans. by NTA (C)
Allen Ans. (C)
Sol. $\quad \mathrm{T}_{\mathrm{r}}=\tan ^{-1}\left[\frac{(\mathrm{r}+2)-(\mathrm{r}+1)}{1+(\mathrm{r}+2)(\mathrm{r}+1)}\right]$
$=\tan ^{-1}(\mathrm{r}+2)-\tan ^{-1}(\mathrm{r}+1)$
$\mathrm{T}_{1}=\tan ^{-1} 3-\tan ^{-1} 2$
$\mathrm{T}_{2}=\tan ^{-1} 4-\tan ^{-1} 3$
$\mathrm{T}_{\mathrm{n}}=\tan ^{-1}(\mathrm{n}+2)-\tan ^{-1}(\mathrm{n}+1)$
$\overline{\mathrm{S}_{\mathrm{n}}=\tan ^{-1}(\mathrm{n}+2)-\tan ^{-1} 2}=\tan ^{-1}\left(\frac{\mathrm{n}+2-2}{1+2(\mathrm{n}+2)}\right)$

$$
=\tan ^{-1}\left(\frac{n}{2 n+5}\right)
$$

$\lim _{n \rightarrow \infty} 6 \tan \left(\tan ^{-1}\left(\frac{n}{2 n+5}\right)\right)$
$=\lim _{n \rightarrow \infty} \frac{6 n}{2 n+5}=\frac{6}{2}=3$
19. Let $\vec{a}$ be a vector which is perpendicular to the vector $3 \hat{i}+\frac{1}{2} \hat{j}+2 \hat{k} . \quad$ If $\vec{a} \times(2 \hat{i}+\hat{k})=2 \hat{i}-13 \hat{j}-4 \hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2 \hat{i}+2 \hat{j}+\hat{k}$ is
(A) $\frac{1}{3}$
(B) 1
(C) $\frac{5}{3}$
(D) $\frac{7}{3}$

Official Ans. by NTA (C)
Allen Ans. (C)
Sol. $\quad(\overrightarrow{\mathrm{a}} \times(2 \hat{\mathrm{i}}+\hat{\mathrm{k}})) \times\left(3 \hat{\mathrm{i}}+\frac{1}{2} \hat{\mathrm{j}}+2 \hat{\mathrm{k}}\right)$
$=(2 \hat{\mathrm{i}}-13 \hat{\mathrm{j}}-4 \hat{\mathrm{k}}) \times\left(3 \hat{\mathrm{i}}+\frac{1}{2} \hat{\mathrm{j}}+2 \hat{\mathrm{k}}\right)$
$-(6+2) \vec{a}=\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k} \\ 2 & -13 & -4 \\ 3 & \frac{1}{2} & 2\end{array}\right|$
$\vec{a}=3 \hat{i}+2 \hat{j}-5 \hat{k}$
Projection of $\vec{a}$ on vector $2 \hat{i}+2 \hat{j}+\hat{k}$ is
$\overrightarrow{\mathrm{a}} \cdot \frac{(2 \hat{\mathrm{i}}+2 \hat{\mathrm{j}}+\hat{\mathrm{k}})}{3}=\frac{5}{3}$
20. If $\cot \alpha=1$ and $\sec \beta=-\frac{5}{3}$, where $\pi<\alpha<\frac{3 \pi}{2}$ and $\frac{\pi}{2}<\beta<\pi$, then the value of $\tan (\alpha+\beta)$ and the quadrant in which $\alpha+\beta$ lies, respectively are
(A) $-\frac{1}{7}$ and $\mathrm{IV}^{\mathrm{th}}$ quadrant
(B) 7 and $\mathrm{I}^{\text {st }}$ quadrant
(C) -7 and $I V^{\text {th }}$ quadrant
(D) $\frac{1}{7}$ and $\mathrm{I}^{\text {st }}$ quadrant

Official Ans. by NTA (A)
Allen Ans. (A)
Sol. $\quad \cot \alpha=1, \sec \beta=\frac{-5}{3}, \cos \beta=\frac{-3}{5}, \tan \beta=\frac{-4}{3}$ $\tan (\alpha+\beta)=\frac{1-\frac{4}{3}}{1+\frac{4}{3} \times 1}=\frac{-1}{7}$

## SECTION-B

1. Let the image of the point $P(1,2,3)$ in the line $L: \frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}$ be $Q$. let $R(\alpha, \beta, \gamma)$ be a point that divides internally the line segment PQ in the ratio $1: 3$. Then the value of $22(\alpha+\beta+\gamma)$ is equal to

Official Ans. by NTA (125)
Allen Ans. (125)

Sol.


Let M be the mid-point of PQ
$\therefore \mathrm{M}=(3 \lambda+6,2 \lambda+1,3 \lambda+2)$
Now, $\overrightarrow{\mathrm{PM}}=(3 \lambda+5) \hat{i}+(2 \lambda-1) \hat{j}+(3 \lambda-1) \hat{k}$
$\because \overrightarrow{\mathrm{PM}} \perp(3 \hat{\mathrm{i}}+2 \hat{\mathrm{j}}+3 \hat{\mathrm{k}})$
$\therefore 3(3 \lambda+5)+2(2 \lambda-1)+3(3 \lambda-1)=0$
$\lambda=\frac{-5}{11}$
$\therefore \mathrm{M}\left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11}\right)$
Since R is mid-point of PM
$22(\alpha+\beta+\gamma)=125$
2. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62 , and their variance is 20 . A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is

Official Ans. by NTA (0)
Allen Ans. (0)
$\sum_{i=1}^{7}\left|x_{i}-62\right|^{2}$
Sol. $\quad 20=\frac{\sum_{i=1}}{7}$
$\Rightarrow\left|\mathrm{x}_{1}-62\right|^{2}+\left|\mathrm{x}_{2}-62\right|^{2}+\ldots .+\left|\mathrm{x}_{7}-62\right|^{2}=140$
If $\mathrm{x}_{1}=49$
$|49-62|^{2}=169$
then,
$\left|x_{2}-62\right|^{2}+\ldots .+\left|x_{7}-62\right|^{2}=$ Negative Number
which is not possible, therefore, no student can fail.
3. If one of the diameters of the circle $x^{2}+y^{2}-2 \sqrt{2} x$ $-6 \sqrt{2} y+14=0$ is a chord of the circle $(x-2 \sqrt{2})^{2}$ $+(y-2 \sqrt{2})^{2}=r^{2}$, then the value of $r^{2}$ is equal to Official Ans. by NTA (10)

Allen Ans. (10)

## Sol.



PQ is diameter of circle
$S: x^{2}+y^{2}-2 \sqrt{2} x-6 \sqrt{2} y+14=0$
$\mathrm{C}(\sqrt{2}, 3 \sqrt{2}), \mathrm{O}(2 \sqrt{2}, 2 \sqrt{2})$
$r_{1}=\sqrt{6}$
$S_{1}:(x-2 \sqrt{2})^{2}+(y-2 \sqrt{2})^{2}=r^{2}$
Now in $\triangle \mathrm{OCQ}$
$|\mathrm{OC}|^{2}+|\mathrm{CQ}|^{2}=|\mathrm{OQ}|^{2}$
$4+6=r^{2}$
$\mathrm{r}^{2}=10$
4. If $\lim _{x \rightarrow 1} \frac{\sin \left(3 x^{2}-4 x+1\right)-x^{2}+1}{2 x^{3}-7 x^{2}+a x+b}=-2$, then the value of $(a-b)$ is equal to

Official Ans. by NTA (11)
Allen Ans. (11)
Sol. $\lim _{x \rightarrow 1} \frac{\sin \left(3 x^{2}-4 x+1\right)-x^{2}+1}{2 x^{3}-7 x^{2}+a x+b}=-2$
For finite limit
$a+b-5=0$
Apply L'H rule
$\lim _{x \rightarrow 1} \frac{\cos \left(3 x^{2}-4 x+1\right)(6 x-4)-2 x}{\left(6 x^{2}-14 x+a\right)}=-2$
For finite limit
$6-14+a=0$
$a=8$
From (1) $b=-3$
Now $(\mathrm{a}-\mathrm{b})=11$
5. Let for $\mathrm{n}=1,2, \ldots . .50, \mathrm{~S}_{\mathrm{n}}$ be the sum of the infinite geometric progression whose first term is $\mathrm{n}^{2}$ and whose common ratio is $\frac{1}{(\mathrm{n}+1)^{2}}$. Then the value of $\frac{1}{26}+\sum_{\mathrm{n}=1}^{50}\left(\mathrm{~S}_{\mathrm{n}}+\frac{2}{\mathrm{n}+1}-\mathrm{n}-1\right)$ is equal to Official Ans. by NTA (41651)

Allen Ans. (41651)

$$
\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}^{2}}{1-\frac{1}{(\mathrm{n}+1)^{2}}}=\frac{\mathrm{n}(\mathrm{n}+1)^{2}}{(\mathrm{n}+2)}
$$

$S_{n}=\frac{n\left(n^{2}+2 n+1\right)}{(n+2)}$
$\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}[\mathrm{n}(\mathrm{n}+2)+1]}{(\mathrm{n}+2)}$
$S_{n}=n\left[n+\frac{1}{n+2}\right]$
$\mathrm{S}_{\mathrm{n}}=\mathrm{n}^{2}+\frac{\mathrm{n}+2-2}{(\mathrm{n}+2)}$
$\mathrm{S}_{\mathrm{n}}=\mathrm{n}^{2}+1-\frac{2}{(\mathrm{n}+2)}$

Now $\frac{1}{26}+\sum_{n=1}^{50}\left[\left(n^{2}-n\right)-2\left(\frac{1}{n+2}-\frac{1}{n+1}\right)\right]$
$=\frac{1}{26}+\left[\frac{50 \times 51 \times 101}{6}-\frac{50 \times 51}{2}-2\left(\frac{1}{52}-\frac{1}{2}\right)\right]$
$=41651$
6. If the system of linear equations
$2 x-3 y=\gamma+5$,
$\alpha x+5 y=\beta+1$, where $\alpha, \beta, \gamma \in \mathbf{R}$ has infinitely many solutions, then the value of $|9 \alpha+3 \beta+5 \gamma|$ is equal to

Official Ans. by NTA (58)
Allen Ans. (58)

Sol. $2 \mathrm{x}-3 \mathrm{y}=\gamma+5$
$\alpha x+5 y=\beta+1$
Infinite many solution
$\frac{\alpha}{2}=\frac{5}{-3}=\frac{\beta+1}{\gamma+5}$
$\alpha=\frac{-10}{3}, \quad 5 \gamma+25=-3 \beta-3$
$9 \alpha=-30, \quad 3 \beta+5 \gamma=-28$
Now, $9 \alpha+3 \beta+5 \gamma=-58$
$|9 \alpha+3 \beta+5 \gamma|=58$
7. Let $\mathrm{A}=\left(\begin{array}{cc}1+\mathrm{i} & 1 \\ -\mathrm{i} & 0\end{array}\right)$ where $\mathrm{i}=\sqrt{-1}$.

Then, the number of elements in the set
$\left\{n \in\{1,2, \ldots ., 100\}: A^{n}=A\right\}$ is
Official Ans. by NTA (25)
Allen Ans. (25)
Sol. $\quad \mathrm{A}=\left[\begin{array}{cc}1+\mathrm{i} & 1 \\ -\mathrm{i} & 0\end{array}\right]$
$A^{2}=\left[\begin{array}{cc}1+i & 1 \\ -i & 0\end{array}\right]\left[\begin{array}{cc}1+i & 1 \\ -i & 0\end{array}\right]$
$A^{2}=\left[\begin{array}{cc}i & 1+i \\ -i+1 & -i\end{array}\right]$
$A^{4}=\left[\begin{array}{cc}i & 1+i \\ -i+1 & -i\end{array}\right]\left[\begin{array}{cc}i & 1+i \\ -i+1 & -i\end{array}\right]$
$A^{4}=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]=I$
$\mathrm{A}^{4 \mathrm{n}+1}=\mathrm{A}$
$\mathrm{n}=1,5,9$, 97
$\Rightarrow$ total elements in the set is 25 .
8. Sum of squares of modulus of all the complex numbers z satisfying $\overline{\mathrm{z}}=\mathrm{iz}^{2}+\mathrm{z}^{2}-\mathrm{z}$ is equal to

Official Ans. by NTA (2)
Allen Ans. (2)
Sol. $z+\bar{z}=i z^{2}+z^{2}$
Consider $\mathrm{z}=\mathrm{x}+\mathrm{iy}$
$2 x=(i+1)\left(x^{2}-y^{2}+2 x y i\right)$
$\Rightarrow 2 x=x^{2}-y^{2}-2 x y$ and $x^{2}-y^{2}+2 x y=0$
$\Rightarrow 2 x=-4 x y$
$\Rightarrow \mathrm{x}=0$ or $\mathrm{y}=\frac{-1}{2}$
Case 1: $x=0 \Rightarrow y=0$ here $z=0$
Case $2: y=\frac{-1}{2}$
$\Rightarrow 4 x^{2}-4 x-1=0$
$(2 x-1)^{2}=2$
$2 x-1= \pm \sqrt{2}$
$\mathrm{x}=\frac{1 \pm \sqrt{2}}{2}$
Here $z=\frac{1+\sqrt{2}}{2}-\frac{i}{2}$ or $z=\frac{1-\sqrt{2}}{2}-\frac{i}{2}$
Sum of squares of modulus of z
$=0+\frac{(1+\sqrt{2})^{2}+1}{4}+\frac{(1-\sqrt{2})^{2}+1}{4}=\frac{8}{4}=2$
9. Let $S=\{1,2,3,4\}$. Then the number of elements in the set $\{f: S \times S \rightarrow S: f$ is onto and $f(a, b)=f(b, a)$
$\geq \mathrm{a} \forall(\mathrm{a}, \mathrm{b}) \in \mathrm{S} \times \mathrm{S}\}$ is
Official Ans. by NTA (37)
Allen Ans. (37)
Sol. $(1,1),(1,4),(4,1),(2,4),(4,2),(3,4),(4,3)$, $(4,4)$ - all have one choice for image.
$(2,1),(1,2),(2,2)$ - all have three choices for image
$(3,2),(2,3),(3,1),(1,3),(3,3)-$ all have two choices for image.

So the total functions $=3 \times 3 \times 2 \times 2 \times 2=72$
Case 1: None of the pre-images have 3 as image
Total functions $=2 \times 2 \times 1 \times 1 \times 1=4$
Case 2 : None of the pre-images have 2 as image
Total functions $=2 \times 2 \times 2 \times 2 \times 2=32$
Case 3 : None of the pre-images have either 3 or 2 as image

Total functions $=1 \times 1 \times 1 \times 1 \times 1=1$
$\therefore$ Total onto functions $=72-4-32+1=37$
10. The maximum number of compound propositions, out of $\mathrm{p} \vee \mathrm{r} \vee \mathrm{s}, \mathrm{p} \vee \mathrm{r} \vee \sim \mathrm{s}, \mathrm{p} \vee \sim \mathrm{q} \vee \mathrm{s}$,
$\sim \mathrm{p} \vee \sim \mathrm{r} \vee \mathrm{s}, \sim \mathrm{p} \vee \sim \mathrm{r} \vee \sim \mathrm{s}, \sim \mathrm{p} \vee \mathrm{q} \vee \sim \mathrm{s}$, $\mathrm{q} \vee \mathrm{r} \vee \sim \mathrm{s}, \mathrm{q} \vee \sim \mathrm{r} \vee \sim \mathrm{s}, \sim \mathrm{p} \vee \sim \mathrm{q} \vee \sim \mathrm{s}$
that can be made simultaneously true by an assignment of the truth values to $\mathrm{p}, \mathrm{q}, \mathrm{r}$ and s , is equal to

Official Ans. by NTA (9)
Allen Ans. (9)
Sol. If we take

| p | q | r | s |
| :---: | :---: | :---: | :---: |
| F | F | T | F |

The truth value of all the propositions will be true.

