JEE-MAIN – JUNE, 2022

(Held On Tuesday 29th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

Mathematics

Test Pattern : JEE-MAIN

Maximum Marks : 120

Topic Covered: FULL SYLLABUS

Important instruction:

1. Use Blue / Black Ball point pen only.
2. There are three sections of equal weightage in the question paper **Physics, Chemistry** and **Mathematics** having 30 questions in each subject. Each paper have 2 sections A and B.
3. You are awarded +4 marks for each correct answer and −1 marks for each incorrect answer.
4. Use of calculator and other electronic devices is not allowed during the exam.
5. No extra sheets will be provided for any kind of work.

Name of the Candidate (in Capitals) ______________________________

Father's Name (in Capitals) ______________________________

Form Number : in figures ______________________________

: in words ______________________________

Centre of Examination (in Capitals): ______________________________

Candidate's Signature: ______________ Invigilator's Signature : ______________

Rough Space

YOUR TARGET IS TO SECURE GOOD RANK IN JEE-MAIN

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FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to:

(A) 1 (B) α

(C) $1 + \alpha$ (D) $1 + 2\alpha$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $x^4 + x^2 + 1 = 0$

$\Rightarrow (x^2 + x + 1)(x^2 - x + 1) = 0$

$\Rightarrow x = \pm\omega, \pm\omega^2$ where $\omega = 1^{1/3}$ and imaginary.

So $\alpha^{1011} + \alpha^{2022} - \alpha^{3033} = 1 + 1 - 1 = 1$

2. Let arg (z) represent the principal argument of the complex number z. The, $|z| = 3$ and $\arg(z - 1) - \arg(z + 1) = \frac{\pi}{4}$ intersect:

(A) Exactly at one point

(B) Exactly at two points

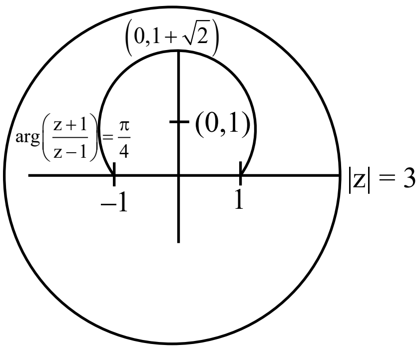
(C) Nowhere

(D) At infinitely many points.

Official Ans. by NTA (C)

Allen Ans. (C)

Sol.



3. Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If $B = I - {}^5C_1\,(\text{adj}A) + {}^5C_2\,(\text{adj}A)^2 - \ldots - {}^5C_5\,(\text{adj}A)^5$, then the sum of all elements of the matrix B is:

(A) –5 (B) –6

(C) –7 (D) –8

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $B = (I - \text{adj}A)^5 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}^5 = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$

Sum of its all elements = –7.

4. The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + \ldots.$ is equal to:

(A) $\frac{425}{216}$ (B) $\frac{429}{216}$ (C) $\frac{288}{125}$ (D) $\frac{280}{125}$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $S = 1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \ldots.$

$\frac{S}{6} = \quad \frac{1}{6} + \frac{5}{6^2} + \frac{12}{6^3} + \frac{22}{6^4} + \ldots.$

---

on subtraction

$\frac{5}{6}S = 1 + \frac{4}{6} + \frac{7}{6^2} + \frac{10}{6^3} + \frac{13}{6^4} + \ldots.$

$\frac{5}{36}S = \quad 1 + \frac{4}{6^2} + \frac{7}{6^3} + \frac{10}{6^4} + \frac{13}{6^5} + \ldots$

---

on subtraction

$\frac{25}{36}S = 1 + \frac{3}{6} + \frac{3}{6^2} + \frac{3}{6^3} + \ldots. = \frac{8}{5}$

$S = \frac{288}{125}$

5. The value of $\lim\limits_{x\to 1}\dfrac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to:

(A) $\dfrac{\pi^2}{6}$ (B) $\dfrac{\pi^2}{3}$ (C) $\dfrac{\pi^2}{2}$ (D) π^2

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $\lim\limits_{x\to 1}\dfrac{(x^2-1)\sin^2\pi x}{(x^2-1)(x-1)^2}=\lim\limits_{x\to 1}\left(\dfrac{\sin((1-x)\pi))}{\pi(1-x)}\right)^2\pi^2=\pi^2$

6. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x)=(x-3)^{n_1}(x-5)^{n_2}$, $n_1, n_2 \in \mathbb{N}$. The, which of the following is <u>NOT</u> true?

(A) For $n_1 = 3$, $n_2 = 4$, there exists $\alpha \in (3,5)$ where f attains local maxima.

(B) For $n_1 = 4$, $n_2 = 3$, there exists $\alpha \in (3,5)$ where f attains local manima.

(C) For $n_1 = 3$, $n_2 = 5$, there exists $\alpha \in (3,5)$ where f attains local maxima.

(D) For $n_1 = 4$, $n_2 = 6$, there exists $\alpha \in (3,5)$ where f attains local maxima.

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $f'(x)=(x-3)^{n_1-1}(x-5)^{n_2-1}(n_1+n_2)\left(x-\dfrac{5n_1+3n_2}{n_1+n_2}\right)$

Option (3) is incorrect since

for $n_1 = 3$, $n_2 = 5$

$f'(x) = 8(x-3)^2(x-5)^4\left(x-\dfrac{30}{8}\right)$

minima at $x=\dfrac{30}{8}$

7. Let f be a real valued continuous function on [0,1] and $f(x)=x+\int_0^1(x-t)f(t)dt$. Then which of the following points (x,y) lies on the curve $y=f(x)$?

(A) (2, 4) (B) (1, 2)

(C) (4, 17) (D) (6, 8)

Official Ans. by NTA (D)

Allen Ans. (4)

Sol. $f(x)=\left(1+\int_0^1 f(t)dt\right)x-\int_0^1 tf(t)dt$

$f(x) = Ax - B$ ...(i)

$A=1+\int_0^1 f(t)dt=1+\int_0^1(At-B)dt$

$\Rightarrow A = 2(1-B)$ ...(ii)

Also $B=\int_0^1 tf(t)dt=\int_0^1(At^2-Bt)dt$

$A=\dfrac{9}{2}B$ ...(iii)

From (2), (3)

$A=\dfrac{18}{13}$, $B=\dfrac{4}{13}$

so $f(6) = 8$

8. If $\int_0^2\left(\sqrt{2x}-\sqrt{2x-x^2}\right)dx=$

$\int_0^1\left(1-\sqrt{1-y^2}-\dfrac{y^2}{2}\right)dy+\int_1^2\left(2-\dfrac{y^2}{2}\right)dy+I$

(A) $\int_0^1\left(1+\sqrt{1-y^2}\right)dy$

(B) $\int_0^1\left(\dfrac{y^2}{2}-\sqrt{1-y^2}+1\right)dy$

(C) $\int_0^1\left(1-\sqrt{1-y^2}\right)dy$

(D) $\int_0^1\left(\dfrac{y^2}{2}+\sqrt{1-y^2}+1\right)dy$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $\text{LHS}=\int_0^2\left(\sqrt{2x}-\sqrt{2x-x^2}\right)dx=\dfrac{8}{3}-\dfrac{\pi}{2}$

$\text{RHS}=\int_0^1\left(1-\sqrt{1-y^2}-\dfrac{y^2}{2}\right)dy+\int_1^2\left(2-\dfrac{y^2}{2}\right)dy+I$

$I+\dfrac{5}{3}-\dfrac{\pi}{4}$

So, $I=1-\dfrac{\pi}{4}=\int_0^1\left(1-\sqrt{1-y^2}\right)dy$

9. If y = y (x) is the solution of the differential equation $\left(1+e^{2x}\right)\frac{dy}{dx}+2\left(1+y^2\right)e^x=0$ and y(0)= 0, then $6\left(y'(0)+\left(y\left(\log_e\sqrt{3}\right)\right)^2\right)$ is equal to:

(A) 2 (B) –2

(C) –4 (D) –1

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $\frac{dy}{1+y^2}+\frac{2e^x}{1+e^{2x}}dx=0$ (i)

on integration

$\tan^{-1}y+2\tan^{-1}e^x=c$

$\because y(0)=0$

so, $C=\frac{\pi}{2}\Rightarrow \tan^{-1}y+2\tan^{-1}e^x=\frac{\pi}{4}$

from eq.(i), $\left(\frac{dy}{dx}\right)_{x=0}=-1$

$\arg y(\ln\sqrt{3})=-\frac{1}{\sqrt{3}}$

$6\left[y'(0)+(y(\ln\sqrt{3})^2\right]=6\left[-1+\frac{1}{3}\right]=-4$

10. Let P : $y^2 = 4ax$, $a > 0$ be a parabola with focus S.Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line y = 3x + 5 touch the parabola P at A and B. Then the value of a for which A,B and S are collinear is:

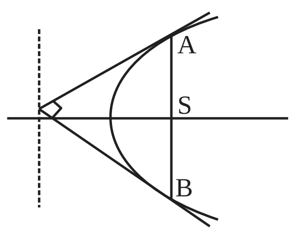
(A) 8 only (B) 2 only

(C) $\frac{1}{4}$ only (D) any a > 0

Official Ans. by NTA (D)
Allen Ans. (D)

Sol. Lines making angle $\frac{\pi}{4}$ with y = 3x + 5 have slope –2 & 1/2.

Which are perpendicular to each-other so, A, S, B are collinear for all a > 0.



11. Let a triangle ABC be inscribed in the circle $x^2 - \sqrt{2}(x+y)+y^2=0$ such that $\angle BAC = \frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the ΔABC is equal to:

(A) $\left(\sqrt{2}+\sqrt{6}\right)/3$ (B) $\left(\sqrt{6}+\sqrt{3}\right)/2$

(C) $\left(3+\sqrt{3}\right)/4$ (D) $\left(\sqrt{6}+2\sqrt{3}\right)/4$

Official Ans. by NTA (Dropped)

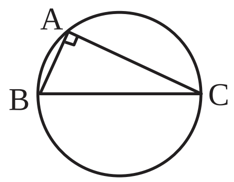
Allen Ans. (Dropped)

Sol. Radius of given circle is 1.

$BC = \text{diameter} = 2,\ AB=\sqrt{2}$

$AC=\sqrt{BC^2-AB^2}=\sqrt{2}$

$\Delta ABC=\frac{1}{2}AB.AC=1$



12. Let $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{-1}$ lie on the plane px – qy + z = 5, for some p, $q\in\mathbb{R}$. The shortest distance of the plane from the origin is:

(A) $\sqrt{\frac{3}{109}}$ (B) $\sqrt{\frac{5}{142}}$

(C) $\sqrt{\frac{5}{71}}$ (D) $\sqrt{\frac{1}{142}}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. (2, –1, –3) satisfy the given plane.

So 2p + q = 8 .... (i)

Also given line is perpendicular to normal plane so

3p + 2q – 1 = 0 .... (ii)

$\Rightarrow$ p = 15, q = –22

Eq. of plane 15x – 22y + z – 5 = 0

its distance from origin $=\frac{6}{\sqrt{710}}=\sqrt{\frac{5}{142}}$

13. The distance of the origin from the centroid of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3},\frac{7}{3}\right)$ is:

(A) $\sqrt{2}$ (B) 2

(C) $2\sqrt{2}$ (D) 4

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $AB \equiv x - 2y + 1 = 0$

$AC \equiv 2x - y - 1 = 0$

So A(1, 1)

Altitude from B is $BH = x + 2y - 7 = 0 \Rightarrow B\ (3, 2)$

Altitude from C is $CH = 2x + y - 7 = 0 \Rightarrow C\ (2, 3)$

Centroid of $\Delta ABC = E(2, 2)$ $OE = 2\sqrt{2}$

14. Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane $x + 2y + 2z = 16$. Let T be a plane passing through the point Q and contains the line $\vec{r} = -\hat{k} + \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right), \lambda \in \mathbb{R}$. Then, which of the following points lies on T?

(A) (2, 1, 0) (B) (1, 2, 1)

(C) (1, 2, 2) (D) (1, 3, 2)

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. Image of P(1, 2, 1) in $x + 2y + 2z - 16 = 0$

is given by Q(4, 8, 7)

Eq. of plane $T = \begin{vmatrix} x & y & z+1 \\ 4 & 8 & 6 \\ 1 & 1 & 2 \end{vmatrix} = 0$

$\Rightarrow 2x - z = 1$ so B(1, 2, 1) lies on it.

15. Let A, B, C be three points whose position vectors respectively are:

$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in \mathbb{R}$

$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$

If α is the smallest positive integer for which $\vec{a}, \vec{b}, \vec{c}$ are non-collinear, then the length of the median, in ΔABC, through A is:

(A) $\frac{\sqrt{82}}{2}$ (B) $\frac{\sqrt{62}}{2}$

(C) $\frac{\sqrt{69}}{2}$ (D) $\frac{\sqrt{66}}{2}$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $\overrightarrow{AB} \parallel \overrightarrow{AC}$ if $\frac{1}{2} = \frac{\alpha - 4}{-6} = \frac{1}{2} \Rightarrow \alpha = 1$

$\vec{a}, \vec{b}, \vec{c}$ are non-collinear for $\alpha = 2$ (smallest positive integer)

Mid-point of BC = $M\left(\frac{5}{2}, 0, \frac{9}{2}\right)$

$AM = \sqrt{\frac{9}{4} + 16 + \frac{9}{4}} = \frac{\sqrt{82}}{2}$

16. The probability that a relation R from {x,y} to {x,y} is both symmetric and transitive, is equal to:

(A) $\frac{5}{16}$ (B) $\frac{9}{16}$

(C) $\frac{11}{16}$ (D) $\frac{13}{16}$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. Total no. of relations = $2^{2\times 2} = 16$

Fav. relation = ϕ, {(x, x)}, {(y, y)}, {(x, x)(y, y)}

{(x, x), (y, y), (x, y)(y, x)}

Prob. = $\frac{5}{16}$

17. The number of values of $a \in \mathbb{N}$ such that the variance of 3, 7, 12 a, 43 – a is a natural number is:

(A) 0 (B) 2

(C) 5 (D) infinite

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. Mean =13

$$\text{Variance } = \frac{9+49+144+a^2+(43-a)^2}{5} - 13^2 \in N$$

$$\Rightarrow \frac{2a^2-a+1}{5} \in N$$

$\Rightarrow$ $2a^2 - a + 1 - 5n = 0$ must have solution as natural numbers

its D = 40n – 7 always has 3 at unit place

$\Rightarrow$D can't be perfect square

So, a can't be integer.

18. From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60°. The pole subtends an angle 30° at the top of the tower. Then the height of the tower is:

(A) $15\sqrt{3}$ (B) $20\sqrt{3}$

(C) $20+10\sqrt{3}$ (D) 30

Official Ans. by NTA (4)

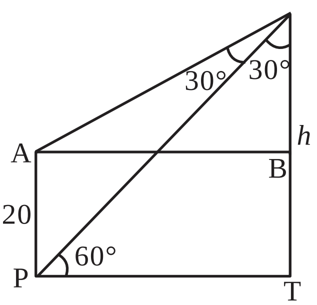
Allen Ans. (4)

Sol. $PT = \frac{h}{\sqrt{3}} = AB$

$$\frac{AB}{h-20} = \sqrt{3}$$

$h = 3(h - 20)$

$h = 30$



19. Negation of the Boolean statement $(p \vee q) \Rightarrow ((\sim r) \vee p)$ is equivalent to:

(A) $p \wedge (\sim q) \wedge r$ (B) $(\sim p) \wedge (\sim q) \wedge r$

(C) $(\sim p) \wedge q \wedge r$ (D) $p \wedge q \wedge (\sim r)$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $P \vee q \Rightarrow (\sim r \vee p)$

$\equiv \sim (p \vee q) \vee (\sim r \vee p)$

$\equiv (\sim p \wedge \sim q) \vee (p \vee \sim r)$

$\equiv [\sim p \vee p) \wedge (\sim q \vee p)] \vee \sim r$

$\equiv [\sim q \vee p) \vee \sim r$

Its negation is $\sim p \wedge q \wedge r$

20. Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\,\alpha$ and $6^n - 5n - 1 = 25\,\beta$, then $\alpha - \beta$ is equal to:

(A) $1 + {}^nC_2\,(8–5) + {}^nC_3\,(8^2 – 5^2) + ... + {}^nC_n\,(8^{n-1} – 5^{n-1})$

(B) $1 + {}^nC_3\,(8–5) + {}^nC_4\,(8^2 – 5^2) + ... + {}^nC_n\,(8^{n-2} – 5^{n-2})$

(C) ${}^nC_3\,(8–5) + {}^nC_4\,(8^2 – 5^2) + ... + {}^nC_n\,(8^{n-2} – 5^{n-2})$

(D) ${}^nC_4\,(8–5) + {}^nC_5\,(8^2 – 5^2) + ... + {}^nC_n\,(8^{n-3} – 5^{n-3})$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $\alpha = \frac{(1+8)^n - 8n - 1}{64} = {}^nC_2 + {}^nC_3 8 + {}^nC_4 8^2 + ...$

$\beta = {}^nC_2 + {}^nC_3 5 + {}^nC_4 5^2 + ...$

option (3) will be the answer.

SECTION-B

1. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} + (\vec{b} \times \vec{c}) = \vec{0}$ and $\vec{b}.\vec{c} = 5$. Then, the value of $3(\vec{c}.\vec{a})$ is equal to____.

Official Ans. by NTA (10)

Allen Ans. (Bonus)

Sol. $\vec{a}+\vec{b}\times\vec{c}=0$

$\vec{a}\times\vec{b}+|\vec{b}|^2\,\vec{c}-5\vec{b}=0$

It gives $\vec{c}=\frac{1}{3}(10\hat{i}+3\hat{j}+2\hat{k})$

so $3\vec{a}.\vec{c}=10$

But it does not satisfy $\vec{a}+\vec{b}\times\vec{c}=0$.

This question has data error.

Alternate (Explanation) :

According to given $\vec{a}\,\&\,\vec{b}$

$\vec{a}\cdot\vec{b}=1-2+3=2$ .... (i)

but given equation

$\vec{a}=-(\vec{b}\times\vec{c})$

$\Rightarrow \vec{a}\perp\vec{b} \Rightarrow \vec{a}\cdot\vec{b}=0$

which contradicts.

2. Let y = y(x), x > 1, be the solution of the differential equation $(x-1)\frac{dy}{dx}+2xy=\frac{1}{x-1}$, with $y(2)=\frac{1+e^4}{2e^4}$. If $y(3)=\frac{e^\alpha+1}{\beta e^\alpha}$. then the value of $\alpha+\beta$ is equal to_____.

Official Ans. by NTA (14)

Allen Ans. (14)

Sol. $\frac{dy}{dx}+\frac{2x}{x-1}.y=\frac{1}{(x-1)^2}$

$y=\frac{1}{(x-1)^2}\left[\frac{e^{2x}+1}{2e^{2x}}\right]$

$y(3)=\frac{e^6+1}{8e^6}$

$\alpha+\beta=14$

3. Let 3, 6, 9, 12,... upto 78 terms and 5, 9, 13, 17,... upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to___.

Official Ans. by NTA (2223)

Allen Ans. (2223)

Sol. For series of common terms

a=9, d=12, n=19

$S_{19}=\frac{19}{2}[2(9)+18(12)]=2223$

4. The number of solutions of the equation sin x = $\cos^2 x$ in the interval (0,10) is__.

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $\sin^2x+\sin x-1=0$

$\sin x=\frac{-1+\sqrt{5}}{2}=+ve$

Only 4 roots

5. For real numbers a,b ($a>b>0$), let

$$Area\left\{(x,y):x^2+y^2\le a^2\text{and }\frac{x^2}{a^2}+\frac{y^2}{b^2}\ge 1\right\}=30\pi$$

and

$$Area\left\{(x,y):x^2+y^2\ge b^2\text{and }\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1\right\}=18\pi$$

Then the value of $(a-b)^2$ is equal to__.

Official Ans. by NTA (12)

Allen Ans. (12)

Sol. given $\pi a^2-\pi ab=30\pi$ and $\pi ab-\pi b^2=18\pi$

on subtracting, we get $(a-b)^2=a^2-2ab+b^2=12$

6. Let f and g be twice differentiable even functions on (–2, 2) such that $f\left(\frac{1}{4}\right)=0, f\left(\frac{1}{2}\right)=0, f(1)=1$ and $g\left(\frac{3}{4}\right)=0, g(1)=2$ Then, the minimum number of solutions of $f(x)\,g''(x)+f'(x)g'(x)=0$ in (–2,2) is equal to__.

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Let $h(x) = f(x)\,g'(x) \to 5$ roots

$\because$ f (x) is even $\Rightarrow$

$$f\left(\frac{1}{4}\right) = f\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right) = f\left(\frac{1}{4}\right) = 0$$

g(x) is even $\Rightarrow g\left(\frac{3}{4}\right) = g\left(-\frac{3}{4}\right) = 0$

$g'(x) = 0$ has minimum one root

$h'(x)$ has at last 4 roots

7. Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}} - \frac{1}{x^{\frac{1}{5}}}\right)^{15}$, $x > 0$, be m and n respectively. If r is a positive integer such $mn^2 = {}^{15}C_r.\,2^r$, then the value of r is equal to__.

Official Ans. by NTA (5)

Allen Ans. (5)

Sol. $T_{r+1} = (-1)^r.{}^{15}C_r.2^{15-r}\,x^{\frac{15-2r}{5}}$

$m = {}^{15}C_{10}2^5$

$n = -1$

so $mn^2 = {}^{15}C_5 2^5$

8. The total number of four digit numbers such that each of the first three digits is divisible by the last digit, is equal to______.

Official Ans. by NTA (1086)

Allen Ans. (1086)

Sol. Let the number is abcd, where a,b,c are divisible by d.

	No. of such numbers
d = 1,	$9 \times 10 \times 10 = 900$
d = 2	$4 \times 5 \times 5 = 100$
d = 3	$3 \times 4 \times 4 = 48$
d = 4	$2 \times 3 \times 3 = 18$
d = 5	$1 \times 2 \times 2 = 4$
d = 6, 7, 8, 9	$4 \times 4 = 16$

1086

9. Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where α is a non-zero real number an $N = \sum_{k=1}^{49} M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is ____.

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$; $M^2 = \begin{bmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{bmatrix} = -\alpha^2 I$

$N = M^2 + M^4 + \ldots\ldots + M^{98} = [-\alpha^2 + \alpha^4 - \alpha^6 + \ldots.]I$

$$= -\alpha^2 \frac{(1-(-\alpha^2)^{49})}{1+\alpha^2}.I$$

$I - M^2 = (1 + \alpha^2)\, I$

$(I - M^2)N = -\alpha^2(\alpha^{98} + 1) = -2$

$\alpha = 1$

10. Let f(x) and g(x) be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$, and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is_____.

Official Ans. by NTA (18)

Allen Ans. (18)

Sol. $f(g(x) = 8x^2 - 2x$

$g(f(x) = 4x^2 + 6x + 1$

So, $g(x) = 2x - 1$ $\quad$ $g(2) = 3$

& $f(x) = 2x^2 + 3x + 1$

$f(2) = 8 + 6 + 1 = 15$

Ans. 18